Advancing Medical Technology,
Aging Population, and Economic Growth

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March 2006

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Executive Summary

Life expectancy in Europe and North-America has more than doubled in the past 200 years—from thirty to thirty-five years at birth in 1800 to seventy-eight years at birth in 2000. Since the last quarter of the nineteenth century, the rise in life expectancy has been accompanied by a downward trend in fertility. This parallel decrease in mortality and fertility rates is known to economists as the “demographic transition.” The demographic transition has coincided with phenomenal technological progress that is continuing and even accelerating. This great wave of a technological progress is reflected in remarkable growth of per capita product (and all other possible measures of material wealth). These demographic and economic trends, however, are uneven in terms of their causes and measures over time.

In the past twenty years, many economic models in the “Modern Growth” literature have offered an analytical framework with which to explain and simulate demographic transition in view of technological progress. A common feature of these models links the decrease in fertility and mortality to technological progress in the following sense: the basic assumption is that parents care about (derive utility from) the number of their children who survive to adulthood and their quality, usually defined in terms of human capital as measured by potential income. As mortality rates decline, it becomes increasingly worthwhile for parents to invest in each child’s human capital and to have fewer children. This linkage among mortality, fertility, and human-capital accumulation may be classified as a version of the well known quantity–quality tradeoff in fertility decisions, formulated back in the 1960s, and implemented widely since then.

The focus on infant and child mortality seems to have been relevant only until the middle of twentieth century, as by then, they already declined to very low levels. During the 19th century it was improved nutrition that lowered infant's mortality rates, achieved by the rising productivity and income along with the economies industrialization. At the first half of the 20th century
mortality rates were cut thanks to the discovery of effective preventive and
curing treatments (sanitation, vaccines etc.) to most of the deadly infective
diseases (such as Malaria, Polio, Dysentery, Typhus, etc.). These reductions in
mortality rates were mainly in those of infants' and young children's, who are
more vulnerable to malnutrition and infections.

In the second half of the twentieth century, however, life expectancy was
extended by another ten years—this time due to significant progress in medical
technology as a direct derivative of overall technological development. This
revolutionary progress in medical technology offered new methods of
diagnosis—MRI, CT, etc.—as well as curative techniques such as heart
surgeries, endoscopies operations, and organ transplants, not to mention a vast
number of pharmaceutical innovations. Technological progress in healthcare,
as in other sectors, has been driven by growing investment in R&D, and to
implement these high-tech novelties, a well-schooled and-trained labor force is
needed. Hence, the utilization of medical technologies and healthcare services
becomes significantly expensive, so the services became available mainly
through the health-insurance market—public and/or private—that developed
around the middle of the twentieth century.

Since the middle of the twentieth century, all industrialized economies
have experienced a steady and significant increase in the share of national
healthcare expenditure in national product. For example, the share of healthcare
expenditure in American GNP rose monotonically from 5% in 1950 to 15% in
2000. The rise in healthcare expenditure has been at the center of the public
political and academic debate. Healthcare expenditure is not distributed equally
among age groups: the older the cohort, the larger its share in expenditure
relative to its share in the population. Most countries subsidize, to some extent,
the use of healthcare services by the elderly, most of whom are retirees. The
concurrence of rising longevity and falling fertility is aggravating the burden of
this intergenerational support, especially in the countries that have Pay As You
Go healthcare systems.
The present study offers a theoretical macroeconomic model for this most recent phase of demographic and economic developments that have characterized modern industrialized countries in the past five decades, stressing the effects of advancing medical technology in these dynamics. The model is ought to use as a comprehensive framework for the analysis of health tax policies in the context of economic growth and demographic dynamics. In the model, Endogenous accumulation of human capital increases labor productivity and promotes technological progress in the medical industry. Technological progress lowers the relative price of health services. It is shown that the rising income and decreasing price of health services allow the elderly to prolong their life expectancy by using increasing amounts of healthcare services—but not necessarily by consuming a larger share of healthcare expenditure. As adults invest more in their human capital, they bear fewer children. Thus, the aging of the population is two-tailed. We characterize the optimal health tax rate, and analyze the affects of suboptimal taxation on the dynamics of growth and aging.
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Abstract  

Endogenous accumulation of human capital increases labor productivity and promotes technological progress in the medical industry. Technological progress lowers the relative price of health services. The rising income and decreasing price of health services allow the elderly to prolong their life expectancy by using increasing amounts of healthcare services—but not necessarily by consuming a larger share of healthcare expenditure. As adults invest more in their human capital, they bear fewer children. Thus, the aging of the population is two-tailed. We characterize the optimal health tax rate, and analyze the affects of suboptimal taxation on the dynamics of growth and aging.

1. Introduction  

Life expectancy in Europe and North-America has more than doubled in the past 200 years—from thirty to thirty-five years at birth in 1800 to seventy-eight years at birth in 2000. Since the last quarter of the nineteenth century, the rise in life expectancy has been accompanied by a downward trend in fertility, from 4.8 average births per woman in 1870 to 1.4 in 1995 (Livi-Bacci, 2000). This parallel decrease in mortality and fertility rates is known to economists as the “demographic transition.” The demographic transition has coincided with phenomenal technological progress that is continuing and even accelerating (Mokyr, 1990). This great wave of technological progress is reflected in remarkable growth of per capita product (and all other possible measures of material wealth). These demographic and economic trends, however, are uneven in terms of their causes and measures over time.
The first ten-year gain in life expectancy was achieved during the nineteenth century. The fruits of the Industrial Revolution—a significant increase in per-capita income and consumption—led to a great improvement in nutrition (and other basic elements of standard of living, such as clothing), which in turn improved (physical) labor productivity (Fogel, 1993). The improvement in health, occasioned by the much better nutrition that resulted in lower mortality rates—especially for infants and children, who are more vulnerable to malnutrition—had nothing to do with medical science (Mokyr, 1993).

The next twenty-year gain in life expectancy was achieved in the first half of the twentieth century. Around the beginning of that century, preventive medicines for most bacterial diseases (malaria, cholera, typhus, etc.) were found. In the decades that followed, “classic” forms of public healthcare—vaccines and antibiotics, disinfection of water, hygienic education, etc.—were developed and implemented. The near-eradication of main infectious diseases marked a great contribution by medical science and technology to mortality reduction, especially (again) for infants, whose immune systems are more vulnerable. Furthermore, these preventive medicines did not incur a significant cost at the household and public level. This, however, would change in the following fifty years.

In the second half of the twentieth century, life expectancy was extended by another ten years—this time due to significant progress in medical technology as a direct derivative of overall technological development, mainly via computer-related innovations. This revolutionary progress in medical technology offered new methods of diagnosis—MRI, CT, etc.—as well as curative techniques such as heart surgeries and organ transplants, not to mention a vast number of pharmaceutical innovations. These technological breakthroughs and improvements powered much of the life-expectancy extension, which originated mainly in the reduction of mortality rates among adults and the elderly (Cutler and Meara, 2001). To implement these high-tech novelties, a well-schooled and-trained labor force is needed.

Technological progress in healthcare, as in other sectors, has been driven mainly by growing private investment in R&D. As a property right, once technologies become more widely implemented and their use requires more advanced professional training, the utilization of medical technologies and healthcare services becomes
significantly expensive in terms of direct monetary cost\(^1\) (Newhouse, 1992). As the growing cost to user of healthcare services subjected use to severe uncertainty, the services became available mainly through the health-insurance market that developed around the middle of the twentieth century (Weisbrod, 1991).

Currently, health-insurance services may be provided as public insurance services that are funded on a “Pay As You Go” (PAYG) basis, e.g., in Canada for instance; in private insurance markets as is common in the U.S.; or in some mixture of the two. Since the middle of the twentieth century, all industrialized economies have experienced a steady and significant increase in the share of national healthcare expenditure in national product. For example, the share of healthcare expenditure in American GNP rose monotonically from 5% in 1950 to 15% in 2000. The rise in healthcare expenditure has been at the center of the public political and academic debate. Healthcare expenditure is not distributed equally among age groups: the older the cohort, the larger its share in expenditure relative to its share in the population. Most countries subsidize, to some extent, the use of healthcare services by the elderly, most of whom are retirees. The concurrence of rising longevity and falling fertility is aggravating the burden of this intergenerational support, especially in the countries that have PAYG healthcare systems.

The present study focuses on this most recent phase of demographic and economic developments that have characterized modern industrialized countries in the past five decades, stressing the effects of advancing medical technology in these dynamics.

Let us place the present study in the context of the literature that deals already with the different phases and aspects of the demographic transition.

\(^1\) Although much of the new medical knowledge and many of its techniques are implemented at the household production level—healthier diet, abstinence from tobacco, and other healthy lifestyle attributes—the significant direct cost of health improvements and lifespan prolongation comes from the consumption of healthcare services—medical care, drugs etc.—that are technology-intensive.
2 Review of the Literature

2.1 Infant/Child Mortality, Fertility Choice, and Growth

In the past twenty years, many economic models in the “Modern Growth” literature have offered an analytical framework with which to explain and simulate demographic transition in view of technological progress. A common feature of these models links the decrease in fertility and mortality to technological progress in the following sense: the basic assumption is that parents care about (derive utility from) the number of their children who survive to adulthood and their quality, usually defined in terms of human capital as measured by potential income. As mortality rates decline, it becomes increasingly worthwhile for parents to invest in each child’s human capital and to have fewer children. This linkage among mortality, fertility, and human-capital accumulation may be classified as a version of the well known quantity–quality tradeoff in fertility decisions, formulated back in the 1960s, and implemented widely since then, (see for example by Ben Porath, 1976; Barro and Becker, 1989; Becker, Murphy, and Tamura, 1990; Galor and Weil, 1996, 2000). In early models in this literature, the reduction of mortality rates at birth was assumed to be exogenous to the parents; in later models it became endogenous, inducing an inertial growth dynamic. When infant mortality is endogenous, the mortality rate of infants and children decreases as parents’ income increases, in a way that may be interpreted as an income effect on nutrition or the supply of public healthcare services. (See, for example, Kalemli-Ozcan, Ryder, and Weil, 2000; and Azarnert, 2005.)

According to the historical survey presented in Section 1, the focus on child mortality seems to have been relevant until the middle of twentieth century. By then, infant mortality probabilities had already declined to very low levels. Also as mentioned above, during the last fifty to sixty years, the prolongation of life expectancy in industrialized countries involved the adoption of new medical technologies that were used mainly (in terms of expenditure) to prevent and cure morbidity in adults. During these decades, however, fertility rates have continued to fall—below replacement level—and technological progress has continued.
2.2 Models of Adult Longevity and Growth

Exogenous Adult Longevity

In recent years, there has been a growing interest in understanding the effects of changes in adult health and life expectancy on the dynamics of growth, capital accumulation (physical and human), and fertility choices. Ehrlich and Lui (1991) may have been the first to investigate the relationship among adult longevity, fertility, and growth. Assuming that children support their parents at old age, they analyze the choice of the quality-quantity tradeoff and optimal investment in children's human capital in view of exogenous changes in infant mortality and parent longevity.

Other works that investigate adult longevity came along more recently. Soares (2005), for example, investigates the effect of a decline in child and adult mortality on fertility, human capital accumulation and growth when parent are altruistic toward their children and derive direct utility from the extended longevity of their offspring. Soares finds that investment in both child and parent human capital increases in tandem with longevity while the fertility rate decreases.

Capriani and Blackburn (2002) model the negative effects of adults’ longevity on fertility (without considering children’s quality) and its positive effect on adults’ investment in their own human capital as the working horizon is extended. In this study, the mortality of adults is determined by their parents’ education. In Chakraborty (2004) the decline in adult mortality promotes growth by promoting adults’ savings and investment in education (with no reference to fertility-demography issues). In this study, mortality is a function of investment in health capital financed by a fixed public health tax on income.²

In both of the last-mentioned studies, an increase in longevity raises the expected return on investment in young adulthood in terms of a longer working life—i.e., a longer stream of return (income)—and the dynamic analysis in both studies focuses on the possibilities of becoming locked in poverty traps characterized by short life expectancy and low investment in human capital (i.e., they focus on early stages of technological and economic development).

² Although the tax rate is fixed throughout the dynamic analysis, the possibility of optimizing it by the social planner is considered in a footnote. Basic modeling of longevity as a function of a public health tax on income also appears in Blackburn and Capriani (2002).
Endogenous Adult Longevity

Two recent unpublished papers model endogenous longevity in a manner that resembles the way it done in our study. According to Finlay (2005), the increase in life expectancy is endogenous to the agent who invests resources (part of income) in an implicit health-enhancing activity and self-protection, which increase the probability of surviving to the second period of working life. The study focuses on the effects of the donation of healthcare services to a poor country on the probability of escaping poverty traps. Finlay finds that investment in healthcare services is a necessary condition for investment in education but that donated healthcare services may crowd out private local investments in healthcare wherever donated education services stimulate private local investment in both education and healthcare.

Bhattacharya and Qiao (2005) study the optimal mix of private and public financing of healthcare, with the public and private investments as complementary inputs in the survival function. They show that, according to some parameters, a public health tax enhances social welfare even though it may expose the economy to chaotic fluctuations. This study does not take investment in human capital into account. The investment in healthcare increases longevity, savings, and consumption after retirement.

None of these works contains a fertility choice or an explicit modeling of the healthcare production sector. Thus, the investment in healthcare is modeled as a forgone consumption good (i.e., the relative price of consumption and health services is equal one).

All the models we discussed so far, except Soares (2005), use the overlapping generations (OLG) frames work. Another exceptional study by Van Zon and Muysken (2001) uses the framework of Lucas’ (1988) infinite horizon growth model with constant fertility, to investigate economic growth in the presence of health technology that extends longevity and increases the productivity of human capital. Focusing on a steady-state analysis, they conclude that the preference of healthcare may result in slow growth rates associated with an aging population and high per-capita product.3

3 The defense economics literature follows a related line of research that studies the macroeconomic effects (growth, savings etc.) of changes in mortality rates due to terror or war activity. (See, for example, Eckstein and Tsiddon 2004, and Nir, 2004, who use the OLG framework as we do in the present work. However, there are at list two main distinctions between health and violence mortality and mortality prevention: 1) while health mortality increases with age, violence mortality seems to be neutral to age; 2) defense services are naturally public goods while health services are utilized at the private level.
Several papers focus on the link between medical technology progress and healthcare expenditure. Jones (2002) stresses the role of medical technology progress in determining the share of healthcare expenditure in OECD countries, focusing on the large expenditure that occurs during the last years of life due to the use of the most expensive technologies to treat severe (deadly) illnesses. In this study, medical technology that advances at an exogenous rate makes it possible to cure more severe diseases and prolong life expectancy. Technological progress affects costs in two opposite directions: cutting-edge technologies are more costly while the old ones become cheaper in absolute terms. In this work income grows at an exogenous rate as well.

Healthcare expenditure, which is most intensive among the elderly, is financed both by private savings and by transfers from the young in a complementary Pay As You Go paradigm. The extent of the transfers is determined in view of an exogenous social willingness to support the old. Social preferences regarding the amount of intergenerational support establish the limits of the increasing share of national health expenditure in product.

Simulations of the model yield a good fit to twentieth-century data (in the U.S.) and, assuming that current transfers have reached the limit of society’s willingness to pay, the projection is for a mild decrease in the share of healthcare expenditure due to the decrease in the price of healthcare services.

In a subsequent study, however, Jones and Hall (2004) claim that medical technology progress per se is not the main source of the increase in healthcare expenditure. Instead, it is a kind of income effect: as general technology progresses, income increases and the marginal utility of consumption decreases. Thus, extending the horizon of consumption (achieving greater longevity) makes it possible to put the growing income to greater use. Jones and Hall characterize the optimal share of healthcare expenditure as the ratio of utility elasticity to healthcare-production elasticity. Their analysis is based on the assumption that the elasticity of utility with respect to consumption decreases faster than the elasticity of healthcare (longevity) production with respect to healthcare-service utilization. They use a calibrated
simulation to demonstrate that under plausible assumptions about the utility function—its elasticity with respect to consumption—one may obtain a monotonic increase in optimal expenditure on healthcare up to 30% of income (until the middle of the century). The model is multi-periodic and multigenerational, and the decisions about healthcare expenditure are made by a social planner. As in the previously mentioned study, the growth rates of income and technology are exogenously assumed. Both studies assume fixed fertility rates.

3. **The Present Study**

The present study models endogenous dynamics of growth, two-tailed population aging and advancing medical technology. We introduce into the conventional fertility–growth framework the choice of extending adult life expectancy. To analyze the aging process of wealthier industrialized economies in which the elderly are retired, we focus on medical technology progress that prolongs the lives of the elderly after retirement.

By explicitly modeling the healthcare-services production sector and its endogenous technological progress, we may analyze the evolution of healthcare expenditures and some aspects of health taxation within a general equilibrium framework. In contrast to the models of Van Zon and Muysken (2001), Jones (2002), Jones and Hall (2004), and Soares (2005), ours is not a multi-periodic model but a two-period overlapping generations model. Thus, instead of extending longevity by additional periods, we increase life expectancy in terms of the probability of surviving into the second period of life (as in Finlay, 2005, and Cachunbary, 2004, for example). This bounds the ability to prolong life to two full periods (i.e., certainty of survival into the second period).

This structural feature of the model, in our opinion, embodies the limited ability of advancing medical technology to extend life expectancy due to the biological aging process. Biological aging is a genetic process of sorts, in which most vital functions deteriorate continuously (from around the age of thirty), making the aging body more vulnerable to all kinds of illnesses and malfunctioning that eventually result in death (Bengston and Warner, 1999). Thus, the older people are, the more caregiving and curative treatment they need to achieve a marginal prolongation of life. The biological
lifespan of the human being—commonly perceived as constant—is not likely to be affected by health technology. The documented limit of longevity is about 120 years, irrespective of the progress in medical technology during the twentieth century.\(^4\) From this perspective, health technology progress enables people to utilize more out of the biological lifespan but at decreasing marginal returns. By subjecting the productivity of health technology to the bounds of biological aging, we take an approach that is somewhat contrary to that of Jones and Hall (2004).\(^5\) Therefore, we obtain contrary results, showing that optimal healthcare expenditure eventually decreases as technological progress advances.

The model can be used as a comprehensive framework for the analysis of health tax policies in the context of economic growth and demographic dynamics. Some preliminary results regarding aspects of health taxation are presented.

The rest of the paper is organized as follows: Section 2 presents the formal model; Section 3 constructs the static optimization conditions; Section 4 introduces the dynamic equilibrium of the model with some health taxation implications; Section 5 concludes the present study; and Section 6 establishes foundations for future research.

### 4. The Model

In an overlapping generations economy with a homogeneous population, agents consume, reproduce (bear children), and rear their children. Adults work in the production of consumption good and healthcare services, and they generate human capital—their own and that of their children.

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\(^4\) Galor and Moav (2002) claim that the urbanization process was associated with an increase in the biological lifespan as it become preferable, from an evolutionary point of view, to switch from a quantity strategy of reproduction to a quality strategy of reproduction.

\(^5\) As in their explanation and projection of the observed increase in healthcare expenditure, they assume that the elasticity of utility with respect to consumption decreases faster than the elasticity of longevity with respect to healthcare-service utilization.
4.1 Agents

The agents may live for two periods: they live the first period with certainty and they survive into the second period probabilistically. The probability of survival depends on the use of healthcare services.

Agents enter their first (active) period of life with initial basic human capital that their parents created while raising them. During their first period of life, agents allocate their time among (a) investment in education (occupational training), (b) bearing and rearing their offspring, and (c) working.

Education raises adult's human capital and thus the productivity of working time. The time devoted to rear each child positively affects child's basic human capital – child's quality. Working income is allocated between the consumption of healthcare services (at the end of the first period) and saving for consumption in the second period.\(^6\) The amount of healthcare services utilization positively affects the probability of surviving into the second period of life (resembling life expectancy).

The second period of life is devoted (probabilistically) to consumption. In the case of death, savings are forgone (i.e., not returned by the lenders). Assuming consumption in two periods or a perfect annuities market should not affect the qualitative results.

The agents’ preferences over consumption and the quantity and quality of children are represented by the following separable expected utility function:

1) \[ E[U(c,n,h)] = \rho \cdot \pi(z) \cdot u(c) + u(n \cdot h) \quad u' > 0, u'' < 0 \]

The variables are defined as follows:

- \( \rho \) — time discount factor
- \( \pi(z) \) — survival probability as a function of healthcare-service utilization — \( z \)
- \( c \) — consumption
- \( n \) — number of children
- \( h \) — the human capital of each child.

\(^6\)This kind of timing greatly simplifies the dynamic analysis of the model by avoiding intergenerational trade in the market for healthcare services. Obviously it is unrealistic, since most healthcare expenditure accrues late in life. However, this is the only way the current literature analyzes healthcare-service utilization. In the last section of this paper, we analyze a case of intergenerational exchange in the market for healthcare services.
4.2 Production

Household Production and Labor Supply

Workers are given one unit of time in their working (first) period of life. They allocate the time between educational attainment, denoted by $e$, and raising their children.

Giving birth to each child incurs fixed time cost $F$; raising each child incurs a variable cost of $v$, to be chosen by the parent. The fixed cost of bearing a child reflects the finite nature of the number of births that a parent can give during the fertile period of her life.

Thus, the time that a worker who has $n$ children can devote to labor activity is $1 - e - n \cdot (F + v)^7$. The effective labor supply of a worker is the product of the time devoted to work and his/her human capital, which is denoted by $g$. Thus, a worker’s supply of labor time is:

\[ l = g \cdot (1 - e - n \cdot (F + v)) \]

The worker’s total human capital $g$ is a function of her basic human capital - acquired from her parent as a child - denoted by $h$, and her chosen educational attainment, denoted by $e$. We specify total human capital as follows:

\[ g(h, e) = h \cdot e^\lambda \quad (0 < \lambda \leq 1) \]

The production of each child’s basic human capital consumes quantity $v$ of parent time and complementary parent education, subject to the standard Cobb-Douglas form:

\[ h_i(v_{i-1}, e_{i-1}) = e_{i-1}^{1-\gamma} \cdot v_{i-1}^\gamma \quad (0 < \gamma < 1) \]

The foregoing production function of basic human capital is uncommon in the literature of fertility choice. It reflects the internalization of the conventional positive externality of parent education on the productivity of offspring.

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\[ ^7 \text{For the sake of convenient we omit the time index wherever it is unnecessary.} \]
Market Production

Consumption product, denoted by $C$, and healthcare services, denoted by $Z$, are produced in a two-sector economy. Each sector uses labor and capital as factor inputs with neoclassic production technology that implies constant returns to scale. Both sectors are perfectly competitive. Labor input is measured in terms of effective labor units, which are the product of labor (time) supply and workers’ human capital.

Each sector has a different productivity factor, determined by the average level of education in the economy. Thus, the productivity factors vary according to the evolution of the educational level in the economy. When the educational level rises, technological progress takes place (as in the conventional literature on modern growth).

We specify the production function of each sector in the Cobb-Douglas form:

\begin{align*}
5a) \quad C &= a(\bar{\sigma}) \cdot L_e^\alpha \cdot K_e^{1-\alpha} \\
5b) \quad Z &= b(\bar{\sigma}) \cdot L_z^\beta \cdot K_z^{1-\beta}
\end{align*}

where:

- $L$ — aggregate effective unit of labor input
- $K$ — capital
- $a(\bar{\sigma}), b(\bar{\sigma})$ — productivity factors, which are functions of the average level of education in the economy — $\bar{\sigma}$.

We assume that technological progress is faster in the health sector than in the consumption sector. Thus, if education follows a rising path, the relative price of healthcare services will decline. We model the decline in the relative prices of healthcare services in order to capture the observed fact that the development, discovery, and adoption of new healthcare technologies allows consumers to purchase services that were previously unavailable at any price. Thus, the price of life prolongation in terms of all other resources in such a process declines.8

We also assume that healthcare services are not tradable internationally. This assumption relies on the perception that healthcare is based on labor-intensive services. After all, there is no substitute for the skilled labor of doctors and nurses in making an accurate diagnosis, performing surgeries (and other medical procedures

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8 The same approach is applied by Jones and Hall (2004).
using advanced technology), and writing correct prescriptions. This assumption is supported by the composition of healthcare expenditure. Indeed, the cost of drugs (and other tradable healthcare goods) is not the main component of healthcare expenditure despite the great technological development that has occurred in the pharmaceutical industry. Most healthcare expenditure is devoted to payment for labor inputs.

By also assuming that the economy is small and open (with a fixed interest rate), we obtain the relative price of healthcare services and income \( w \), as a function of the average level of education. Income is defined by:

\[
6) \quad w = \tilde{A} \cdot a(\bar{e},) \frac{1-\alpha}{\alpha} \cdot l
\]

where \( \tilde{A} \) determined by the parameters of the production functions and the interest rate. To focus on the technological progress that has occurred in the healthcare sector, we assume that consumption sector has constant technology, thus:

\[
6a) \quad \tilde{A} \cdot a(\bar{e},) \frac{1-\alpha}{\alpha} = A \Rightarrow w = A \cdot l
\]

The relative price of healthcare services in terms of consumption goods is:

\[
7) \quad p_e = p(\bar{e},)
\]

For later simulations, we specify the following price function:

\[
7a) \quad p_e(\bar{e},) = 0.5^{m \bar{e}}, \quad (m > 0)
\]

This price function was chosen in order to satisfy the condition of a price that decreases with the level of education, at a decreasing rate—\( p'(\bar{e}) < 0, p''(\bar{e}) > 0 \). The parameter \( m \) resembles the sensitivity of healthcare-technology progress to the level of education. One may think of an increase in \( m \) as an improvement in healthcare-production technology that is exogenous to the level of education—a akin to the “unexpected” discovery of new major treatment in view of a fixed level of education. Figure 1 shows the price function (for \( m = 15 \)) with the level of education on the horizontal axis and the price of healthcare services on the vertical axis.

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9 Only 10% of total health expenditure in the U.S in 2000 went for drugs (Cerda, 2004).
10 See the full elaboration in appendix 1.
The production of health itself—measured in life expectancy—is subject to survival function $\pi(z) \in [\mu, 1]$, which increases with the utilization of healthcare services but with a declining marginal return:

\begin{align*}
\text{(8)} & \quad \text{for } z_i = 0; \quad \pi(0) = \mu \quad \text{where } 0 < \mu < 1, \\
& \quad \text{for } z_i > 0; \quad \mu < \pi(z_i) < 1, \\
& \quad \text{and } : \quad \pi'(z_i) > 0 \quad \pi''(z_i) < 0 \quad \lim_{z_i \to 0} \pi(z_i) = 1.
\end{align*}

The parameter $\mu$ is the base survival probability that the agent has if s/he uses no healthcare services at all. Changes in the base survival probability may account for changes in life expectancy that are exogenous to the use of healthcare services, e.g., improvements in lifestyle, environmental conditions, or any other health-related parameters that are cost-free.

The assumed form of the survival function is common for the modeling of life expectancy within an OLG framework.\(^{11}\) Bear in mind that this way of modeling life expectancy bounds the maximum longevity. We interpret the bound as biological due to genetic nature of the aging process. According to this interpretation, the use of healthcare services helps to prevent death due to illnesses within the biological lifespan but becomes more and more costly as the aging process progresses.

Later in this study we will use the following particular specification:

\(^{11}\) See, for example, Cachunbary (2004) and Finlay (2005).
8a) \( 0 < \mu < 1 \quad \pi(z) = \mu + \frac{(1-\mu) \cdot z}{z+1} \)

### 5. Agent Optimization

Putting together all the characters of the economy we have defined, the representative agent should solve the following constrained optimization:

\[
\begin{align*}
9) \quad \max_{c,n,v,z} & E\left[U(c,n,v)\right] = \rho \cdot \pi(z) \cdot u(c) + u(n \cdot h(v,e)) \\
\text{s.t.} & c = (w-p \cdot z) \cdot (1+r) \\
& w = A \cdot (1-e-n \cdot (F+v)) \cdot h_{t-1} \cdot e^\gamma \\
& e + n(F+v) \leq 1 \\
& n,v \geq 0.
\end{align*}
\]

Using the aforementioned constraints, we may reformulate the maximization problem in terms of the following indirect utility function:

\[
9a) \quad \max_{c,n,v,z} \left\{ E[U(c,n,v)] \right\} = \rho \cdot \pi(z) \cdot u(1+r) \cdot A \cdot (1-e-n \cdot (F+v)) \cdot h_{t-1} \cdot e^\gamma - p \cdot z \}
\]

Differentiating the four chosen variables, we obtain the following first-order conditions:

1. Differentiating for \( e \), we obtain:

\[
10) \quad \rho \cdot \pi(z) \cdot u'(c) \cdot (1+r) \cdot \left[ h_{t-1} \cdot A \cdot e^\gamma \left( -\frac{\lambda}{e} + (1+\lambda) + \frac{\lambda}{e} \cdot n \cdot (v+F) \right) \right] = \\
\]

Optimum level of education \( e \) should equalize the marginal effect of education on the utility of consumption occasioned by increasing income to its effect on the marginal utility of children due to the positive effect on children’s human capital.

2. Differentiating for \( n \), we get:

\[
11) \quad \rho \cdot \pi(z) \cdot u'(c) \cdot (1+r) \cdot A \cdot h_{t-1} \cdot (F+v) \cdot e^\gamma = e^{1-\gamma} v^\gamma \cdot u(n \cdot e^{1-\gamma} v^\gamma)
\]
The optimum number of children equalizes the marginal increase in utility from the marginal child to the marginal loss of utility from consumption due to the time devoted to each child, which has an opportunity cost in working time, i.e., income.

Dividing Condition 1 by Condition 2, we get:

\[
12) \quad n = \frac{\lambda - e \cdot (1 + \lambda)}{((\lambda + \gamma - 1) \cdot (v + F)\gamma)}
\]

**Proposition 1:** The number of children is a decreasing function of parent's educational attainment, fixed cost of giving birth, total time invested in each child, and the productivity of time investment in the formation of basic human capital.

The outcome of Proposition 1 resembles the competitive uses of time over the fertile and productive years of adult life, expressed in the tradeoff that a parent faces between increasing her own income (i.e., consumption) and having more children. It also resembles the quality–quantity tradeoff, as an increase in parents’ education, time invested in each child and its productivity has a positive effect on child's quality. The next proposition will define the exact form of this tradeoff.

A positive number of children requires:

\[
e < \frac{\lambda}{1 + \lambda} \quad \text{and} \quad \lambda + \gamma > 1, \quad \text{or} \quad e > \frac{\lambda}{1 + \lambda} \quad \text{and} \quad \lambda + \gamma < 1.
\]

Computing the agent’s income after plugging in the optimal number of children, we obtain:

\[
13) \quad w = A \cdot h_{r-1} \cdot e^A \cdot \left(\frac{(\gamma - 1) + (2 - \gamma) \cdot e}{(\lambda + \gamma - 1)}\right)
\]

Hence, for the wage to be positive, \( e \) should satisfy \( e > \frac{(1 - \gamma)}{(2 - \gamma)} \) and \( \lambda + \gamma > 1 \).

3. Differentiating for \( v \), we get:

\[
14) \quad \rho \cdot \pi(z) \cdot u'(c) \cdot (1 + r) \cdot A \cdot h_{r-1} \cdot n \cdot e^A = \gamma \cdot n \cdot e^{1 - \gamma} \cdot v^{r - 1} \cdot u'(n \cdot e^{1 - \gamma} \cdot v^r)
\]
Optimizing the time invested in each child equalizes the marginal utility of the increase in the child’s human capital to the marginal loss of utility due to forgone income and consumption.

Combining conditions 2 and 3, we obtain:

15) \( (F + v) = \frac{v}{\gamma} \Rightarrow v = \frac{F}{1 - \gamma} \)

**Proposition 2:** The time invested in each child is fixed by the parameters \( F \) and \( \gamma \).\(^{12}\)

Proposition 2 specifies the quality–quantity tradeoff that was introduced in proposition 1, as an 'indirect tradeoff' only: the number of children declines with the parent’s education level, which, in turn, increases the productivity of the time investment in each child’s human capital (quality) while the time invested in each child is fixed. However as parent's education is higher the fixed time invested in each child bears higher opportunity cost in terms of foregone income.

Two parameters affect positively the optimal amount of time invested. (a) The productivity of time in producing child’s human capital has a positive effect on the returns to investment and, thereby, on the optimal investment level as well. (b) The fixed cost of bearing a child makes the quantity costly relative to quality. Before moving on, we use the foregoing results to obtain utility \( u(v, n, e) \) as a function of the parent’s educational attainment only:

\[
(16) \quad u(n, e, v) = u(e) = \left[ \frac{\lambda - e \cdot (1 + \lambda)}{(\lambda + \gamma - 1) \cdot (v + F)} \cdot e^{\lambda \gamma} \cdot \left( \frac{\gamma \cdot F}{1 - \gamma} \right) \right] = \left[ (\lambda \cdot e^{\lambda \gamma} - (1 + \lambda) \cdot e^{2\gamma \gamma}) \cdot G \right]
\]

where: \( G = \left( \frac{\gamma \gamma}{\lambda + \gamma - 1} \right) \cdot \left( \frac{1 - \gamma}{F} \right)^{1-\gamma} \).

4. Differentiating for \( z \), we obtain:

17) \( \pi'(z) \cdot u(c) = (1 + r) \cdot p \cdot \pi(z) \cdot u'(c) \)

\(^{12}\)This result holds for other conventional specifications of human capital functional forms, such as:

\( h(v, e) = (\theta + e) \cdot v, \quad (\theta > 0) \), as long as \( e \) is canceled in the ratio: \( v = \frac{h(v, e)}{h'(v, e)} \).
This first-order condition requires agents to equalize the benefit from utilizing a marginal unit of healthcare service (on the left) to its opportunity cost (the expected marginal utility of consumption). Equation (17) may be translated into terms of elasticity:

\[
\pi'(z) = \frac{(1+r) \cdot p \cdot u'(c)}{u(c)} = \frac{\eta_{\pi,z}}{\pi(z)} = \frac{\varepsilon_{u,c}}{c} \Rightarrow \frac{\eta_{\pi,z}}{(1+r) \cdot p \cdot z} = \frac{\varepsilon_{u,c}}{(1+r) \cdot c} \Rightarrow \frac{\eta_{\pi,z}}{\varepsilon_{u,c}} = \frac{p \cdot z}{c}
\]

where \( \eta_{\pi,z} \) is the elasticity of life expectancy with respect to the utilization of healthcare services and \( \varepsilon_{u,c} \) is the elasticity of utility with respect to consumption\(^{13}\).

Equation (17a) states that the optimal ratio of the shares of expenditure (on healthcare and consumption) is equal to the ratio of the elasticity of the survival function with respect to healthcare services and the elasticity of utility with respect to consumption.

To derive the demand for healthcare services, we present Equation (17) in the following form:

\[
\pi'(z) = \frac{u(c)}{u'(c)} = \frac{(1+r) \cdot p \cdot \pi(z)}{\pi'(z)}
\]

To obtain an explicit function of the agent’s demand for healthcare services, we apply the survival function defined in Equation (8a):

\[
\pi(z) = \mu + \frac{(1-\mu) \cdot z}{z+1}, \quad (0 < \mu < 1)
\]

We also assume that the utility function takes the conventional CIES (CRRA) form: \( u(c) = \frac{c^{1-\phi}}{\phi} \) and, for convenience, we set \( \phi = \frac{1}{2} \) so that the utility function is \( u(c) = 2\sqrt{c} \). The assumed utility function has constant elasticity with respect to the level of consumption, \( \varepsilon_{u,c} = 2 \).

Figure 2 shows that the elasticity of the assumed survival function (vertical axis) increases (decreases) with low (high) level of healthcare utilization (horizontal axis).

\(^{13}\) Same result has derived by Hall and Jones (2004).
First, elasticity increases quickly with \( z \) and then decreases relatively slowly, verging on zero as \( z \) approaches infinity.

![Figure 2](image-url)

The assumed functional forms clash with Hall and Jones’ (2004) assumption that the elasticity of the utility function decreases more quickly than the elasticity of healthcare production. This difference in assumptions is reflected in the different results that we obtain regarding the evolution of the share of healthcare expenditure: According to Equation (17a), and to the specific functional forms that we assume, the share of healthcare expenditures will rise at a low level of \( z \) and will fall at a high level of \( z \). Plugging the assumed specific functions into Equation (17b), we obtain\(^{14}\):

\[
2 \cdot c = (1 + r) \cdot p \cdot \frac{(\mu + z)(z + 1)}{(1 - \mu)}
\]

Using the identity \( c = (w - p \cdot z) \cdot (1 + r) \),\(^ {15}\) we obtain:

\[
z = \sqrt[2]{w \cdot \frac{2 \cdot (1 - \mu)}{p} + \frac{(3 - \mu)^2}{4} - \mu - \frac{(3 - \mu)}{2}}
\]

**Proposition 3:** Demand for healthcare services and life expectancy are a non-decreasing function of real income, expressed in terms of health.

---

\(^{14}\) Note that \( \pi'(z) = \frac{(1 - \mu)}{(z+1)^2} \), thus:

\[
\pi(z) = \frac{(\mu + z)(z + 1)}{(1 - \mu)}
\]

\(^{15}\) See full elaboration in Appendix 2.
More specifically, as long as real income—in terms of healthcare services—is below a certain minimum level, demand for healthcare services will be zero (and life expectancy equals $\mu$). Above this level the demand for healthcare increases with income and decreases with healthcare price. The minimum income level rises with the base survival rate, as defined in the following inequality:

$$19a) \quad \frac{w}{p} > \frac{\mu}{2 \cdot (1 - \mu)}$$

Since healthcare services appear to be a normal good under the specified functional forms of the utility and survival function, the share of these services rises (falls) with income for low (high) levels of income.

Putting propositions 1 and 3 together, we find that the educational process is linked to two-tailed aging as a consequence of adult agent optimization. Higher education is acquired in order to sustain higher income, which accommodates both a higher level of consumption and a longer horizon of consumption at old age. Education and the bearing and rearing of children are the competing uses of the productive and fertile age interval. Thus, the higher (expected) utility of consumption that is obtained by means of higher education is followed by a reduction in the time devoted to children, resulting in less utility of children due to lower fertility. The negative effect of lower fertility on the utility derived from children is partly compensated for by an increase in the quality of children due to the positive external effect of their parent’s education.

6. **Dynamics of Growth and Aging Population:**

6.1 **Existence of Converging Dynamics**

Using the results obtained in the first-order optimization conditions, we rewrite agent utility as a function of a single-choice variable—the level of education, $e$. The optimal education level should maximize the following utility function:

$$20) \quad \max_e E[U] = \rho \cdot \pi(z(e)) \cdot u((1 + r) \cdot [w(e) - p(\bar{e}) \cdot z(e)]) + u(m(e) \cdot h(e))$$
Plugging the explicit functions $z(e), n(e), h(e), \text{and } w(e)$ into Equation (20) and differentiating for $e$, we obtain a single first-order condition that defines the solution for the maximization problem of the agents:

21) \[
\rho \cdot \left[ \pi'(z) \cdot z' \cdot u(c) + \pi(z) \cdot u'(c) \cdot c' \right] + \nabla \cdot \left( (1-\gamma) \lambda \cdot e^{-\gamma} - (2-\gamma)(1+\lambda) \cdot e^{1-\gamma} \right) \cdot v' \left( (1+\lambda) \cdot e^{1-\gamma} - (1+\lambda) \cdot e^{2-\gamma} \right) \cdot G = 0.
\]

By using Equation (17) — $\pi'(z) \cdot u(c) = (1+r) \cdot p \cdot \pi(z) \cdot u'(c)$ — and setting $\rho$ to be equal $(1+r)^{-1}$, we may simplify the foregoing expression to:

21a) \[
\frac{\pi(z) \cdot u(c)}{\sigma} \cdot w + \nabla \cdot \left( (1-\gamma) \lambda \cdot e^{-\gamma} - (2-\gamma)(1+\lambda) \cdot e^{1-\gamma} \right) \cdot v' \left( (1+\lambda) \cdot e^{1-\gamma} - (1+\lambda) \cdot e^{2-\gamma} \right) \cdot G = 0.
\]

By assuming a rational expectations equilibrium in the economy, we impose that the average level of education in the economy — $\bar{e}$ — is equal to the optimal level of education chosen by the representative agent — $e$. Thus, each agent takes into account the expected decrease in the price of healthcare services if the level of education rises. This consideration is not explicit in Equation (21a) but is implicit in the computed optimal amount of $z$ (and $c$) out of any possible (chosen) income.

We express Equation (21a) as $\sigma + \psi \cdot \xi = 0$. Since $\sigma$ and $\xi$ are always positive, a necessary condition for an interior solution is $\psi < 0$, which means:

$$e > \frac{(1-\gamma) \cdot \lambda}{(2-\gamma)(1+\lambda)}.$$

For this condition to be met, the total labor income defined in Equation (12) must increase with education. However, it is already contained in the non-negative income constraint. By imposing the positive income and the number of children, we obtain lower and upper bounds for the optimal level of education, denoted respectively as $e_l$ and $e_u$. Thus, for $\lambda + \gamma > 1$ we obtain:

$$e_l = \frac{(1-\gamma)}{(2-\gamma)} < e^* < \frac{\lambda}{1+\lambda} = e_u.$$
Defining Equation (21a) as \( \phi(e, e_{t-1}) \), we can use the implicit functions theorem to investigate the possible dynamics of the education level—the effect of \( e_{t-1} \) on \( e_t \). As education approaches its lower bound \( (e \rightarrow e_l) \), the right-hand side of Equation (21a) and the entire expression in (21a) approach infinity \( (\sigma \rightarrow \infty) \). By the same token, as education approaches its upper bound \( (e \rightarrow e_u) \), the left-hand side of Equation (21a) and the entire expression in (21a) approach minus infinity \( (\psi \cdot \xi \rightarrow -\infty) \). Hence, if there is a unique interior solution for the optimization problem, it must be that the expression in Equation (21a) decreases as \( e \) approximates the optimal level of education—\( e^* \). In other words, assuming the existence of a unique interior solution, we obtain:

\[
\frac{d\phi(e, e_{t-1})}{de_i} < 0.
\]

Differentiating \( \phi(e, e_{t-1}) \) for \( e_{t-1} \) we obtain:

\[
\frac{d\phi(e, e_{t-1})}{de_{t-1}} = \pi'(z) \cdot z'_{e_{t-1}} \cdot u'(c) \cdot w'_c + \pi(z) \cdot u''(c) \cdot c_{e_{t-1}}' \cdot w'_c + \pi(z) \cdot u'(c) \cdot w_{e_{t-1}, e_{t-1}}.
\]

The first term is positive: \( \pi'(z) \cdot z'_{e_{t-1}} \cdot u'(c) \cdot w'_c > 0 \). Hence, a sufficient condition to insure that \( \frac{d\phi(e, e_{t-1})}{de_{t-1}} > 0 \) is the following:

22) \[ \pi(z) \cdot (u''(c) \cdot c_{e_{t-1}}' \cdot w'_c + u'(c) \cdot w_{e_{t-1}, e_{t-1}}) > 0 \Leftrightarrow u''(c) \cdot c_{e_{t-1}}' \cdot w'_c < u'(c) \cdot w_{e_{t-1}, e_{t-2}} \]

Under the assumed utility and income functions, Condition (22) may be rewritten as:

22a) \[
\frac{1}{2} \cdot c_{e_{t-1}}' \cdot w'_c < c \cdot w_{e_{t-1}, e_{t-1}} \Rightarrow \frac{c_{e_{t-1}}'}{c} < 2 \cdot \frac{w_{e_{t-1}, e_{t-1}}}{w'_c} \Rightarrow \frac{c_{e_{t-1}}'}{c} < 2 \cdot \frac{(1-\gamma)}{e_{t-1}}
\]

\[
\zeta_{e_{t-1}} < 2 \cdot (1-\gamma) \Leftrightarrow \frac{c_{e_{t-1}}'}{c} < 2 \cdot (1-\gamma).\]

Since \( e_{t-1} \) has an upper bound—\( e_u \)—the foregoing condition may be satisfied if the consumption level is high enough. The level that accomplishes this may be attained at a productivity factor—\( A \)—that is large enough. If the condition
\[
\frac{d\phi(e_t, e_{t-1})}{de_{t-1}} > 0 \quad \text{is satisfied, assuming a single interior solution} \quad \left(\frac{d\phi(e_t, e_{t-1})}{de_t} < 0\right),
\]

we find that \(e_t\) increases with \(e_{t-1}\).

### 6.2 Simulations

To demonstrate the existence of the increasing and converging path of the education level, we perform a simulation. The results of the simulation, presented in Figure 3, illustrate the existence of an increasing and converging level of education that defines the optimal paths of all other variables. Life expectancy, consumption, and demand for healthcare services increase while fertility decreases. The simulation yields a fast convergence; it takes about five periods for the variables to reach the steady-state level.

![Figure 3](image)

The decrease in the share of healthcare expenditure deserves special attention because it contradicts the observed trend in real data. At lower values of the education

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16 For the simulation, we use the price function \(-p(\tilde{e})\), specified in Equation 5a.

17 The parameter values we use are: \(\mu = 0.6, r = 0.05, \lambda = 1, \gamma = 0.8, F = 0.05, A = 11, m = 11, e_0 = e_t\).

We use the same parameters in all further simulations. The qualitative results of this simulation are stable for a large range of values around this baseline set.
productivity parameters $A$ and $m$, demand for healthcare is zero and education sustains a low-level steady state. From some threshold of these parameters, the levels of demand for healthcare services and education jump to a high-level steady state. For the present set of parameterization, it happens where $m$ and $A$ approximate the value of 10. In the neighborhood of this critical value, the utility function $U$ does not have a single peak.\footnote{Appendix 4 illustrates the non-concavity of $U$.} Around this threshold level of the parameter values, different paths of healthcare expenditure share are possible. For parameter values that are slightly above the threshold, the share of healthcare expenditure rises monotonically with education along the convergence path. At slightly higher values, the share of healthcare expenditure increases sharply and then decreases moderately, and for higher values the share of healthcare expenditure along the convergence path declines monotonically, as Figure 3 shows. The higher the base survival rate, the higher the threshold level of parameters $A$ and $m$.

The following figures illustrate the different converging paths of healthcare expenditure for parameter values that slightly exceed the critical values (while the convergence path of the other variables is qualitatively unchanged). For the parameters $m = 10, A = 11$ in Figure 3a, the share of healthcare expenditure increases monotonically; for the parameters $m = 10.5, A = 11$ in Figure 3b, the share increases sharply at first and decreases moderately later on.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3a.png}
\caption{Figure 3a}
\end{figure}

\footnote{Appendix 4 illustrates the non-concavity of $U$.}
As we recall (from Equation 17a), the optimal share of healthcare expenditure is (positively) correlated to the elasticity of the survival function with respect to $z$, which first increases and then decreases. For low (high) levels of education productivity (defined by $A$ and $m$), healthcare-service utilization ($z$) is relatively low (high) for either income or substitution effects and lies within the increasing (decreasing) range of elasticity. Thus, the share of healthcare expenditure may rise or decline while the levels of education and healthcare-service utilization increase. This result derive from the assumption that the elasticity of healthcare production (at some point) decreases (faster than the elasticity of consumption utility).

If we reinforce the technological progress by allowing an occasional exogenous increase in the parameter $A$ and $m$, we can make sure that the optimal share of healthcare expenditure will eventually decrease. In view of the points that we have demonstrated, we may identify the trend of healthcare expenditure in the past fifty years as the first phase of development just after reaching the threshold level. Thus, we may expect future technological developments to cause the share of healthcare expenditure to decline.

However, the effect of such a decrease in $m$ on the optimal level of the education path is ambiguous. As we recall, agents derive utility from both consumption and children in accordance with the way they allocate their time, where the level of education reflects the substitution between the two (higher education being associated with higher consumption and fewer children). Thus, a decrease of $m$ induces two
opposite effects. On the one hand, it makes education more productive in terms of life expectancy (and expected utility of consumption); this is a sort of substitution effect. On the other hand, the higher productivity of education has a positive income effect on both sources of utility—consumption and children.

At low values of real income, the elasticity of healthcare production increases with the reduction of $m$ and yields a strong improvement in education productivity, which means a strong substitution effect that dominates the income effect. Consequently, education rises and fertility falls. As the elasticity of healthcare production decreases (with higher values of real income) the substitution effect weakens and the income effect becomes dominant. Thus, the level of education falls and fertility rises even though life expectancy and consumption increase. An increase in $m$ reduces the share of healthcare expenditure at every positive level of healthcare-service utilization.

For an increase in productivity parameter $A$, we found a monotonic positive effect on the optimal education level (i.e., the substitution effect is always dominant). Thus, one may expect future major developments in medical technology, which will significantly improve the benefits gained from health expenditure, to offset (at least partly) the current trend of declining fertility. Figures 4 and 5 present the simulated education levels at steady state (on the vertical axis) for different values of $m$ and $A$ (on the horizontal axis). The threshold levels of the parameters (around the value of 10) are easy to identify in these figures.
An increase in both $A$ and $m$ increases the steady state share of healthcare expenditure in a small neighborhood around their threshold levels, but higher values of $A$ and $m$ are associated with lower share of healthcare expenditure at the steady state\textsuperscript{19}.

Finlay (2005) demonstrates a negative effect of donated healthcare aid to poor countries (which increases adult life expectancy) on the countries’ incentives to invest in education. We obtain a similar result by raising the base survival rate. The base survival rate has a negative effect on the marginal productivity of healthcare services, which inhibits investment in education. However, it also has a positive effect on the (marginal) expected utility of consumption, which stimulates investment in education.\textsuperscript{20} Figures 6 and 6a show that for the set of parameters that we use education and the share of healthcare expenditure at steady state decrease monotonically at the level of the base survival rate. If the base survival rate is high enough, education and the share of healthcare expenditure fall steeply to a low-level steady state.

\textsuperscript{19} See graphs in Appendix 3.

\textsuperscript{20} It is a commonly found in exogenous mortality models that an increase in adults’ survival probability stimulates education investment. See, for example, Blackburn and Capriani (2002).
7. **Expenditure on Healthcare and (sub) Optimal Health-Tax.**

In many developed countries (e.g., Canada, U.K., Sweden, and Israel), the healthcare sector is largely public and financed by a flat-rate income tax. If the healthcare sector is purely public, the health-tax rate equals the share of healthcare expenditure in income.\(^{21}\) In this case, the agent maximizes his utility in view of the following constraints:

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\(^{21}\) In Chakraborty (2004) this kind of health tax serves is the sole source of finance for healthcare expenditure.
The first three optimization conditions that yield propositions 1 and 2 (regarding the optimal number of children and the optimal investment in each child) do not change and the fourth condition (regarding the optimal demand for healthcare services) is no longer relevant because income allocation between consumption and health is given by $\tau$. Thus, we move to the following utility function in the level of education:

$$
23a) \quad Max: E[U] = \rho \cdot \pi[\tau(e)] \cdot \sqrt{(1 + r) \cdot (1 - \tau) \cdot w(e)} + \sqrt{\pi(e)} \cdot h(e) = \\
= \rho \cdot \pi \left[ \frac{\tau \cdot w(e)}{p(\tilde{e})} \right] \cdot \sqrt{(1 + r) \cdot (1 - \tau) \cdot w(e)} + \sqrt{n(e)} \cdot h(e),
$$

It is easy to verify that the optimal tax rate, chosen by a social planner to maximize the utility of agents in each generation, will coincide with the share of healthcare expenditure (in national income) that is optimally chosen by the agents in the decentralized economy.\(^{22}\) Hence it is of interest to investigate the effects of various inefficient tax rates.

Usually there is room for both public and private provision of healthcare services, although the private sector may be strongly regulated and severely restricted in size and in the types of activities allowed to it.\(^{23}\) As long as an overall supply shortage exists, the effect of a suboptimal tax rate will be the same as it would be in the case of a purely public healthcare sector. To obtain over utilization of healthcare services, however, the compulsory public supply must be higher than optimal, resulting in zero demand for private healthcare services.

\(^{22}\) Differentiating Equation 22a for $\tau$, we obtain $\rho \cdot \pi(z) \cdot u(c) = (1 + r) \cdot p(\tilde{e}) \cdot u'(c)$

\(^{23}\) We assume that private and public services are perfect substitutes.
Assuming that total utility function $U(e)$ has a single peak, the effects of (small) deviations from the optimal tax rate are as follows:

**Proposition 4:** tax rates that are higher (lower) than the optimum will raise (lower) the level of education when $\eta$ increases in $e$ and have the opposite effect when $\eta$ decreases in $e$.

Recall that the optimal share of healthcare expenditure as a function of education level follows a bell curve and corresponds to the shape of the elasticity of the survival function. Concavity of $U(e)^{24}$ is sufficient (although not necessary) to ensure that, under the restriction of a non-optimal tax rate, the corresponding restricted optimal level of education will be on the same side of the optimal healthcare-expenditure curve. In other words, the economy does not shift from the decreasing range of $\eta$ to its increasing range (and vice versa).

Thus, for economies on a rising path of healthcare expenditure, an excessive rate of health tax stimulates investment in education and accelerates the dynamic of growth. In economies that are on a downward path, in contrast, suboptimal taxation may accelerate the convergence to steady state. Figure 7 shows simulation results of the education level in steady state (vertical axis) at different levels of health tax (horizontal axis). Note that the optimal share of healthcare expenditure for the set of parameters we use is around 19% and is on a downward slope of $\eta$ (and share of healthcare expenditure).

![Figure 7](image)

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24 I.e., the parameters are out of threshold level's neighborhood.
The proposed (non-monotonic) effect of suboptimal health-tax rate on educational attainment and growth may have interesting implications for income inequality, where a uniform (fixed) health tax is implemented in the presence of population heterogeneity with respect to initial income, human capital, etc. however this issue is beyond the scope of the present study.

8. Concluding Remarks

We have developed a model of endogenous growth, progress in medical technology, and population aging, driven by human capital accumulation. The model allows us to simulate and analyze remarkable economic and demographic phenomena in recent decades. Specifically, it creates an analytical framework for the study of a two-tailed aging population and the growth process in view of advancing medical technology, with a variable share of healthcare expenditure in national income.

The model is able to reproduce the dynamics that are observed as a result of the optimization problem regarding the number and quality of children, life expectancy, and consumption level, reduced to a single-choice variable: the level of education. By using specific functional forms, we could derive explicit demand for healthcare services and demonstrate the evolution and convergence of the optimal path of education, fertility, life expectancy, and healthcare expenditure.

We showed that the assumption of an upper bound for the ability of medical technology to prolong life results in a potential decrease in the share of healthcare. We also showed that a further improvement in the state of healthcare, or a reduction in the price of healthcare services that are exogenous to the level of education, may offset the trends of declining fertility and increasing investment in human capital. To gauge the preliminary implications of the model as an analytical tool for health-tax policy, we explored the effects of non-optimal health-tax rates on rates and levels of growth in steady state.
Appendix 1: Price and wage equations

Using equations (5a) and (5b) and assuming a small open economy, we obtain the following labor-to-capital ratio for each sector:

\[ C = d(\bar{e})L_c^\alpha \cdot K_c^{1-\alpha} = d(\bar{e}) \cdot (N_c \cdot l)^\alpha \cdot K_c^{1-\alpha} \Rightarrow MP_k = (1-\alpha) \cdot d(\bar{e}) \cdot \left( \frac{l}{k_c} \right)^\alpha = r \]

\[ Z = b(\bar{e})L_z^\beta \cdot K_z^{1-\beta} = N_z \cdot l^\beta \cdot K_z^{1-\beta} \Rightarrow MP_k = p \cdot (1-\beta) \cdot b(\bar{e}) \cdot \left( \frac{l}{k_z} \right)^\beta = r \]

where \( k_i \) is capital per worker in the sector, \( i = (c, z) \), and the marginal productivity in the health sector is expressed in terms of a consumption good (i.e., multiplied by relative price \( p \)). Since both sectors share the same interest rate, we obtain:

\[ p = \frac{(1-\alpha) \cdot d(\bar{e}) \cdot l^{\alpha-\beta} \cdot k_z^\beta}{(1-\beta) \cdot b(\bar{e}) \cdot k_c^{\alpha}} \]

Wages in each sector are equal to the marginal productivity of the respective sectors’ labor:

\[ \omega_c = \alpha \cdot a(\bar{e}) \cdot \left( \frac{k_c}{l} \right)^{1-\alpha} \Rightarrow w_c = \alpha \cdot a(\bar{e}) \cdot l^\alpha \cdot k_c^{1-\alpha} \]

\[ \omega_z = p \cdot \beta \cdot b(\bar{e}) \cdot \left( \frac{k_z}{l} \right)^{1-\beta} \Rightarrow w_z = p \cdot \beta \cdot b(\bar{e}) \cdot l^\beta \cdot k_z^{1-\beta} \]

The general equilibrium requires wage equity across sectors:

\[ p = \frac{\alpha \cdot a(\bar{e}) \cdot l^{\alpha-\beta}}{\beta \cdot b(\bar{e})} \cdot \frac{k_c^{1-\alpha}}{k_z^{1-\beta}} \]
Equalizing the two price equations, we find that capital per worker in the healthcare sector is a constant proportion of the capital-per-worker ratio in the consumption sector:

$$\frac{k_c}{k_z} = \frac{(1-\alpha) \cdot \beta}{(1-\beta) \cdot \alpha} \Rightarrow k_z = k_c \cdot D$$

Plugging the last equality into the first price equation, we obtain:

$$p = \frac{(1-\alpha) \cdot a(\bar{e})}{(1-\beta) \cdot b(\bar{e})} \left( \frac{l}{k_c} \right)^{\alpha-\beta}$$

Using the first interest rate—$MPk_c$—condition to express $\frac{l}{k_c}$ in terms of the parameters of the model and the interest rate, we obtain:

$$MPk_c = (1-\alpha) \cdot a(\bar{e}) \left( \frac{l}{k_c} \right)^{\alpha} = r \Rightarrow \frac{l}{k_c} = \left[ \frac{r}{(1-\alpha) \cdot a(\bar{e})} \right]^{\frac{1}{\alpha}}$$

Plugging the above term of $\frac{l}{k_c}$ into the last price equation, we obtain:

$$p(\bar{e}) = \frac{(1-\alpha)^{\beta} \cdot D^\beta \cdot a(\bar{e})^\beta}{r^{\frac{\beta-1}{\alpha}} \cdot (1-\beta) \cdot b(\bar{e})} = \frac{(1-\alpha)^{\beta} \cdot \beta^\beta \cdot a(\bar{e})^\beta}{r^{\frac{\beta-1}{\alpha}} \cdot \alpha^\alpha \cdot (1-\beta)^{\beta} \cdot b(\bar{e})} = H \cdot \frac{a(\bar{e})^\beta}{b(\bar{e})}$$

By plugging the same term for $\frac{l}{k_c}$ into the first wage equation, we may define income from labor as:

$$\omega = \alpha \left( \frac{k_c}{l} \right)^{1-\alpha} = \alpha \cdot \left[ \frac{a(\bar{e})(1-\alpha)}{r} \right]^{(1-\alpha)} \Rightarrow w = \alpha \cdot \left[ \frac{a(\bar{e})(1-\alpha)}{r} \right]^{(1-\alpha)} \cdot l = A \cdot a(\bar{e})^{\frac{1-\alpha}{\alpha}} \cdot l$$
Appendix 2: Elaborating the demand for healthcare services- $z$

From Equation 17a, we obtain:

$$(1 + r) \cdot p \cdot \frac{(\mu + z)(z + 1)}{(1 - \mu)},$$

which may be written explicitly as:

$$2 \cdot (1 + r) \cdot (w - p \cdot z) = (1 + r) \cdot p \cdot \frac{(\mu + z)(z + 1)}{(1 - \mu)}.$$

or, after simple manipulations:

$$\frac{w}{p} = z + \frac{(\mu + z)(z + 1)}{2 \cdot (1 - \mu)}.$$

Elaborating the right side, we obtain:

$$\frac{w}{p} = \frac{z \cdot 2 \cdot (1 - \mu) + (\mu + z) \cdot (z + 1)}{2 \cdot (1 - \mu)} = \frac{z \cdot 2 - z \cdot 2 \cdot \mu + z \cdot \mu + \mu + z^2 + z}{2 \cdot (1 - \mu)}.$$

Now we reduce the right side:

$$\frac{w \cdot 2 \cdot (1 - \mu)}{p} = z^2 + z \cdot (3 - \mu) + \mu \Rightarrow$$

$$\Rightarrow \frac{w \cdot 2 \cdot (1 - \mu)}{p} = \left[ z + \frac{(3 - \mu)}{2} \right]^2 - \frac{(3 - \mu)^2}{4} + \mu$$

in order to isolate $z$:

$$z = \sqrt{\frac{w \cdot 2 \cdot (1 - \mu)}{p} + \frac{(3 - \mu)^2}{4} - \mu - \frac{(3 - \mu)}{2}}$$
Appendix 3: Optimal share of health services at steady state for different parameter values

The following graphs present the share of healthcare expenditure at steady state for different levels of the parameters $A$ and $m$. Note that when $m$ decreases the share of healthcare expenditure verges on zero, but when $A$ decreases the share of healthcare expenditure converges to a significant positive level.
Appendix 4: Threshold Effect and Non-Concavity of $U$

The graphs below show the jump in the optimal level of education around the critical values of $A$ and $m$ —the threshold effect—and the non-concavity of $U$ at this range. On the horizontal axis is the level of education and on the vertical axis is the utility level, with the level of education imposed to be equal for parents and children (steady state).

$U$ for $m, A = 10$

$U$ for $m, A = 11$
References:


