Employment protection, firm selection, and growth∗

– Job Market Paper –

Markus Poschke†
European University Institute, Florence
August 2006

Abstract

This paper analyzes the effect of firing costs on aggregate productivity growth in a model of growth through selection and imitation. The model is consistent with recent evidence on job turnover, firm heterogeneity, and the contribution of firm entry and exit to aggregate productivity growth. Growth arises endogenously via selection among heterogeneous incumbent firms. It is sustained because entrants imitate the best incumbents. In this framework, firing costs not only induce misallocation of labor, reduce firm value, and discourage entry, but also affect growth through the exit margin by discouraging exit of low-productivity firms. This makes selection less severe and slows down growth. However, exempting exiting firms from firing cost speeds up the exit of inefficient firms and thereby growth. These are the two central and new results of the paper. Firing costs affect growth more strongly in sectors where firms face larger idiosyncratic shocks, e.g. in the service sector. These are the sectors where EU-US growth rate differences are largest. Introducing firing costs of one year’s wages (close to values observed in continental Europe) in a benchmark economy calibrated to the US business (services) sector then implies a growth reduction of 0.1 (0.4) points yearly.

JEL codes: E24, G38, J38, J63, J65, L11, L16, O40

Keywords: endogenous growth theory, firm dynamics, entry and exit, firm selection, experimentation, imitation, firing costs, labor market regulation

1 Introduction

This paper analyzes the effect of labor market regulation on productivity growth, a topic that is much less researched than the impact on the level of productivity or on employment. For this

∗I would like to thank Omar Licandro, Andrea Caggese, Alain Gabler, Andrea Ichino, Claudio Michelacci, Salvador Ortigueira, Josep Pijoan, Morten Ravn, Gilles Saint-Paul, Thijs van Rens, and seminar participants at the EUI, CEMFI, and the XI Workshop on Dynamic Macroeconomics in Vigo for valuable comments and discussions.
†Contact: European University Institute, Economics Department, Via della Piazzuola 43, 50133 Firenze, Italy. e-mail: markus.poschke@iue.it. Tel. +39-348-7701271, Fax +39-055-4685-902.
purpose, a heterogeneous-firm model with endogenous growth is developed. Here, labor market regulation will not only have an effect on the efficiency of the allocation of labor across plants, or on the incentive to work or to search, but will affect the endogenous growth of aggregate productivity through its impact on the market selection process through the entry and exit margins.

Recent empirical research on firm dynamics has highlighted the importance of entry and exit and the heterogeneity of firms and plants. For example, Dwyer (1998) finds that productivity differs by a factor 3 between establishments in the 9th and the 2nd decile of the productivity distribution in the US textile sector. Foster, Haltiwanger and Krizan (2001) (FHK) find that in the Census of Manufactures, more than a quarter of the increase in aggregate productivity between 1977 and 1987 was due to entry and exit. This is even more pronounced in the retail sector, as they find in their (2002) paper Gabler and Licandro (2005) find in a calibration exercise that around half of US post-war productivity growth can be traced to the process of market selection, entry, and exit. Generally, entry and exit contribute more to growth in high TFP growth sectors (Bartelsman et al. 2004), and these were precisely the sectors where Europe lagged US productivity growth in recent years (van Ark, Inklaar and McGuckin 2002, Blanchard 2004).

This paper takes this evidence as a point of departure. In particular, the mechanism of growth through selection and experimentation developed here fits many facts on firm dynamics and thus suggests itself for a heterogeneous-firm growth model. Indeed, the basic model is very similar to the one developed in Gabler and Licandro (2005). In its treatment of firing costs, the analysis here is related to the seminal paper of Hopenhayn and Rogerson (1993), and the more recent ones by Alvarez and Veracierto (2001), Veracierto (2001), and Samaniego (2006). These four all analyze the effect of firing costs on the level of aggregate productivity. However, they employ a setting of exogenous growth and concentrate on the effect on the static efficiency of the allocation of labor. Bertola (1994), conversely, analyzes the effect of hiring and firing costs in an endogenous growth model of growth through variety expansion. In such a setting, firing costs affect entry but not exit, so that the selection effect that will be crucial here cannot arise.

In the model developed here, firms receive idiosyncratic productivity shocks and therefore differ in their productivity and employment. Growth arises and is sustained endogenously through

---

the interaction of selection (among incumbents) and imitation (by entrants). Each period, the least productive incumbents are eliminated, implying that the average productivity of remaining firms grows. Entry sustains growth: Entrants try to imitate firms close to the technological frontier. They do not succeed fully, but on average enter a constant fraction below it. Hence, there is a spillover from incumbents to entrants through the location of the frontier. Its strength depends on how much entry and exit, and thus selection, there is, so growth is driven by both selection and imitation.

In this context, labor market regulation affects the entry and exit incentives of firms, and thereby the engines of growth in this model. It is well-known that firing costs, as one-sided adjustment costs, induce an inaction region in firms’ employment policy when productivity is stochastic. As a consequence, labor is not optimally allocated across firms. In a nutshell, the most productive firms do not employ enough, and declining firms do not lay off quickly enough, relative to the situation without firing costs. The deviation of optimal employment with the firing cost from the frictionless optimum lowers firm value. This leads to less entry and lower growth in Bertola (1994).

In the present paper, however, there is an additional effect through exit and selection. To analyze it, it is crucial to distinguish if exiting firms have to pay firing costs, or are exempt (or can default on them). This was already remarked by Samaniego (2006) in an environment of exogenous growth. The crucial observation is that firing costs have two distinct effects: they are an adjustment cost, and a tax on exit. The latter discourages exit of low-productivity firms, thereby weakening the selection process. This slows down productivity growth through selection and affects the productivity of entrants, since they are now targeting a worse distribution. As a result, growth is lower. When exiting firms are exempt, firing costs lower firm value, thereby promoting exit of low-productivity firms, strengthening selection, and increasing growth relative to the frictionless economy. This effect is likely to be important in practice, given that firm exit contributes positively to aggregate productivity growth in almost all countries (see e.g. Bartelsman et al. 2004). The net effect on welfare will depend on the relative size of the static and the dynamic effects, i.e. if faster growth outweighs consumption losses due to misallocation of labor.

The contribution of this paper lies in these results: Firing costs affect the market selection

---

2In addition, there is a role for experimentation because growth depends on the variance of the productivity distribution of entrants. A higher variance makes high draws more likely, while very low draws will be cut off by subsequent exit anyway. As a result, there is a tradeoff between this dynamic gain and the static loss due to the cost of (possibly excessive) entry. This is being explored in parallel research.
process, by hampering selection when they are charged to exiting firms, or by enhancing it otherwise. Selection could be enhanced by anything that lowers firm value and/or encourages exit. The cleanest instrument for achieving this and moving the economy towards its optimal growth rate is an exit subsidy. Firing costs or similar adjustment frictions (such as e.g. reorganization costs or loss of expertise) reinforce selection. Since this comes at the cost of inducing misallocation of labor, they are a very imperfect instrument towards this goal.

To quantitatively evaluate the impact of labor market regulation on observed differences in productivity growth and in the behavior of entrants, the model is calibrated to the US business sector. Then the effects of introducing firing costs of one year’s wages, close to the level observed in Germany and in many other continental European countries, is evaluated. Results show that charging firing costs only to continuing firms promotes selection and thereby growth. However, this is outweighed by the effect of the misallocation of labor, so welfare falls by an equivalent of 3.8% of permanent consumption. When firing costs are also charged to exiting firms they act as an “exit tax” and slow down growth by 0.1 percentage point, implying a welfare loss of 5.4%. These losses are higher in the service sector. Evidence shows that firms there face a more volatile business environment, so firing costs restrict them more, decreasing growth by 0.3 percentage points.

The paper is organized as follows. In the next section, a simple heterogeneous firm model with growth by selection and experimentation is set up. In Section 3, it is solved for optimal behavior of all agents, equilibrium is defined, an algorithm for calculating it is given, and the determination of the growth rate is discussed. In the following section, the model is calibrated, and in Section 5, the quantitative effects of firing costs are explored. Section 6 concludes.

2 The Model Economy

Time is discrete and the horizon infinite. The economy is populated by a continuum of infinitely-lived consumers of measure one, a continuum of active firms of endogenous measure, a large pool of potential entrants, and a sector of perfectly competitive portfolio firms.

Consumers value consumption and dislike working; this is summarized in the period utility function \( u(\hat{c}_t, n_t) = \ln \hat{c}_t - \theta n_t \). They discount the future using a discount factor \( \beta < 1 \). They can consume or invest in shares \( \hat{a}_t \) of the portfolio firms that pay a net return \( r_t \); wages and the return to the portfolio provide them with income.

The portfolio firms finance investment in the firms in the economy and transfer profits as dividends to shareholders. Since the sector is competitive, they do not make any profits and
return the entire net profits of the production sector to consumers as dividends. Given perfect competition and assuming symmetry, they all hold the market portfolio and pay the same return \( r_t \) on assets. Hence, they can be summarized into one representative portfolio firm.\(^\text{3}\)

**Firms:** Firms produce a numéraire good using labor as their only, variable input, with a positive and diminishing marginal product. To remain active, firms also incur a fixed operating cost \( c_f^t \) each period; this grows over time at the growth rate of wages, \( g \). Moreover, there is an exogenous probability \( \delta \) that a firm’s production facilities break down after a period’s production, forcing the firm to exit; this affects all firms in the same way.

Firms differ in productivity. This arises because each firm receives idiosyncratic productivity shocks; more precisely, its productivity follows a random walk. This is a very simple way of capturing the role of idiosyncratic shocks established by the empirical literature. It also renders the persistence of firm level productivity found in the data.\(^\text{4}\) This production technology can be summarized in Assumption 1 and in the production function

\[
\hat{y}_{it} = \exp(\hat{s}_{it}) \hat{n}_{it}^\alpha, \quad 0 < \alpha < 1, \tag{1}
\]

where \( \hat{y}_{it} \) denotes output of firm \( i \) in period \( t \), \( \exp(\hat{s}_{it}) \) is its productivity level, and \( \hat{n}_{it} \) employment.

**Assumption 1** *Productivity evolves according to*

\[
\hat{s}_{it} = \hat{s}_{i,t-1} + \epsilon_{it}, \tag{2}
\]

*where the innovation \( \epsilon \) is distributed normally with mean zero and variance \( \sigma^2 \).*

**Firing costs:** Adjusting employment is costless in the benchmark case. This will be compared to the case with employment protection legislation (EPL) in the form of firing costs of \( c^n \) times a period’s wages for each worker fired. Two cases can be distinguished, one where firing costs always have to be paid upon firing a worker, including upon exit (denoted by \( 1_{x} = 1 \)), and another one where firing costs only have to be paid if the firm also remains active in the subsequent period; i.e. exiting firms are exempted from firing costs (denoted by \( 1_{wone_{x}} = 0 \)). An active firm’s profit function can then be written in a general way as

\[
\hat{\pi}_{it} = \pi(\hat{s}_{it}, n_{it}, n_{i,t-1}, \hat{w}_t) = \exp(\hat{s}_{it}) \hat{n}_{it}^\alpha - \hat{w}_t \hat{n}_{it} - c_f^t - g(\hat{n}_{it}, \hat{n}_{i,t-1}), \tag{3}
\]

\(^3\)The portfolio firms do not play a large role in themselves, they just serve as a device to abstract from liquidity constraints of firms.

\(^4\)There is no very clear agreement what “persistence” quantitatively means in this context. Dwyer (1996) illustrates some of the difficulties encountered in estimation.
where \( \hat{w}_t \) denotes the period-\( t \) wage and the function \( g(\hat{n}_{it}, \hat{n}_{i,t-1}) \) summarizes firing costs. It is given by

\[
g(\hat{n}_{it}, \hat{n}_{i,t-1}) = c^\theta \hat{w}_t \cdot \begin{cases} 
\max(0, \hat{n}_{i,t-1} - \hat{n}_{it}) & \text{if } I_x = 1, \\
\max(0, \hat{n}_{i,t-1} - \hat{n}_{it}) & \text{if } I_x = 0 \land \hat{n}_{it} > 0, \\
0 & \text{if } I_x = 0 \land \hat{n}_{it} = 0.
\end{cases}
\] (4)

The dependence of \( g(\cdot) \) on previous period’s employment makes the employment choice a dynamic decision when there are firing costs, and implies that a firm’s individual state variables are \((s_{it}, n_{i,t-1})\).

At the end of any period, firms can decide to exit. This is costless in the benchmark case and when exiting firms are exempt from firing costs \((I_x = 0)\); otherwise \((I_x = 1)\), the exiting firm has to cover the firing cost for reducing its workforce from \( \hat{n}_{i,t-1} \) to 0. As shown below, it will be optimal for firms to exit if their productivity is below a certain threshold. With \( I_x = 0 \), this threshold will depend on past employment.

**Entry:** Entering firms have to pay a sunk entry cost \( c^e_t \) that grows at the same rate as output. This can be interpreted as an irreversible investment into setting up production facilities.\(^5\) Entrants try to imitate the best firms in the economy; for the sake of concreteness, assume that they identify the best 1% of firms with the frontier of the economy. Varying this figure does not affect results for the benchmark economy much; firing costs have a stronger effect the larger the fraction of firms in the targeted set. Denote average productivity of the target group with \( s^\text{max}_t \). In practice, entrants are on average less productive then incumbents; for instance, (Foster et al. 2001)report that firms that entered within the last 10 years are on average 99% as productive as incumbents. One possible explanation is that they cannot copy incumbents perfectly due to tacitness of knowledge embodied in these firms. Assumption 2 formalizes this process.

**Assumption 2** Entrants draw their initial productivity \( \hat{s}^0_{it} \) from a normal distribution with mean \( \hat{s}^0_t \) and variance \( \sigma^2_e \). Call its pdf \( f_{\hat{s}^0} \). Moreover,

\[
\hat{s}^0_{it} = s^\text{max}_t - \kappa, \quad \kappa > 0.
\] (5)

Because \( \kappa > 0 \), entrants are on average less productive than the best incumbents. The assumption implies that, as the distribution of incumbents moves rightward, the distribution of entrants tracks it at a constant distance \( \kappa \).

\(^5\)Empirical evidence shows that in practice, a large part of investment is irreversible in the sense that the resale value of assets is very low. This is more pronounced the more specific and the less tangible the asset, and the thinner the resale market. For evidence see e.g. Ramey and Shapiro (1998).
Assumption 2 describes an externality; incumbents’ productivity spills over to entrants. While the selection process will drive growth, this externality will be responsible for sustaining it. It can be interpreted in other ways besides imitation. For instance, entrants’ productivity could be related to the technological and institutional conditions in an economy; these are already captured in the productivity distribution of incumbents.

The intensity of experimentation, parametrized by \( \sigma_e^2 \), is related to growth. A higher \( \sigma_e^2 \) implies that the probability of drawing an extreme, including very high, productivity rises. On the other hand, the larger probability of bad draws means that the entry process will consume more resources, making the net effect ambiguous. For the purpose of this paper, take \( \sigma_e^2 \) as fixed by technology.

Denote the distribution of firms over productivity states \( s \) by \( \tilde{\mu}(s) \equiv M\mu(s) \), where \( M \) is the number of firms, and \( \mu(s) \) is a density function with integral one. The assumption of a continuum of firms that are all independently affected by the same stochastic process, together with the absence of aggregate uncertainty, implies that the aggregate distribution evolves deterministically. As a consequence, although the identity of firms with any \( (s, n^-) \) is not determined, their measure is deterministic. Moreover, the underlying probability distributions can be used to describe the evolution of the cross-sectional distribution.

**Timing:** The structure of the economy implies the following timing. At the end of any period, firms decide if they stay or exit, and potential entrants decide whether to enter. In the beginning of the next period, incumbent firms receive their productivity innovations and entrants draw their initial productivity. All firms pay the fixed operating cost \( c^f_t \), entrants in addition pay the entry cost \( c^e_t \). Firms demand labor, workers supply it, and the wage adjusts to clear the labor market. Production occurs, agents consume, and profits are realized. Firms that reduced labor or exited at the end of the previous period pay the firing cost. After this, the whole process resumes. Hence, the dynamic choices of entry, exit, and employment are all made based on firms’ expectations of future productivity.

**Stationarize the economy:** The analysis will focus on the balanced growth path of this economy. Define this as a situation where output, consumption, wages, and aggregate productivity grow at a constant rate \( g \), the firm productivity distribution shifts up the productivity scale in steps of \( g \), the shape of the firm productivity distribution is invariant, and employment, the

---

6Formally, this follows from the Glivenko-Cantelli Theorem (see e.g. Billingsley 1986). For a more thorough discussion, see Feldman and Gilles (1985) and Judd (1985).
firm employment distribution, the interest rate, the share of entrepreneurs, turnover and other
dynamic characteristics of the firm distribution are constant. The growth rate $g$ is endogenous
and will be derived in Section 3.4. Designate variables of the growing economy with a hat. All
growing variables can be made stationary by adjusting them by their (cumulative) growth rates,
i.e.
\[ z_t = \hat{z}_t e^{-gt} = z \]  
for any variable $z$ growing at rate $g$, $x_t = \hat{x}_t = x$ for any constant variable $x$, and
\[ s_{it} = \hat{s}_{it} - gt \]  
for the firm-level productivity state. This implies that in the stationarized economy, firm pro-
ductivity evolves according to
\[ s_{it} = s_{i,t-1} - g + \epsilon_{it} \]  
Firm productivity now follows a random walk with downward drift (for positive growth rates)
because the whole firm productivity distribution shifts up at rate $g$, so in expectation, firms
fall back by $g$ every period relative to the distribution. The sunk entry cost $c^e$ and the fixed
operating cost $c^f$ are constants now. To simplify notation, drop the time subscript $t$ and denote
next period’s values by a superscript $\cdot^+$ and last period’s values by a superscript $\cdot^-$. In the
remainder, the analysis will be in terms of this stationary equilibrium.

3 Equilibrium

This section starts with the derivation of optimal behavior for all agents. Then, equilibrium is
defined and an algorithm for calculating it is given. Finally, the growth rate is derived, and its
determinants discussed.

3.1 Optimal Behavior

Consumers maximize utility by choosing asset holdings and labor supply. Firms maximize the
expected discounted flow of profits by choosing employment, entry, and exit.

Consumers: The consumer problem is completely standard. Utility maximization yields the
Euler equation
\[ 1 + g = \beta(1 + r_t), \]  

8
where \( g \) is the growth rate of consumption. This implies that the prevailing gross interest rate in the economy is \( 1 + r = (1 + g)/\beta \). Moreover, consumers supply labor in accordance with the first order condition \( c = w/\theta \).

**Employment:** Active firms face a standard dynamic optimization problem. This is particularly simple in the case with no firing costs, since then it is a sequence of static problems, and a firm’s productivity \( s \in S \) is its only state variable. Call labor demand for this case \( n_0(s, w) \). With firing costs, last period’s employment \( n^- \in N \) also becomes a state variable for the firm. Aggregate state variables are the wage \( w \) and the growth rate \( g \), since the latter affects the productivity evolution of all firms. So denote the employment policy for the more general problem by \( n(s, n^-, w, g) \). The associated Bellman equation is

\[
V(s, n^-, w, g) = \max_n \{ \pi(s, n, n^-, w) + \frac{1}{1 + r} \max(E[V(s^+, n, w, g)|s], V^x) \},
\]

where the profit function includes the fixed cost and the adjustment cost of labor, the inner max operator indicates the option to exit, and \( V^x \) denotes the value of exit as defined in (11) below.

This is a standard problem, existence and uniqueness of the value function follow from standard arguments. Two properties carry over from the profit function: The value function is increasing and convex in \( s \) given an \( n^- \), and weakly decreasing in \( n^- \) given \( s \) if there are firing costs. Whereas the employment policy \( n(s, n^-, w, g) \) increases monotonically in \( s \) in the frictionless economy, it features a constant part around \( n^- \) when \( c^n > 0 \). Intuitively, when a firm’s productivity increases a little, it will not immediately raise employment because productivity might fall again, and reducing employment again then would be costly. Analogously, when a firm’s productivity falls slightly, it will not immediately fire workers because productivity might recover and it would have paid the firing cost prematurely. When firms are exempted from paying the firing cost upon exit, firms that suffer a negative productivity shock so large that they are forced to exit will not adjust employment downward immediately, but keep it constant and fire all workers upon exit. So given an \( n^- \), the employment policy is constant for \( s \) very low or around \( n^- \), and strictly increasing elsewhere. The employment policy function and the law of motion for \( s \) jointly define a transition function \( Q : S \times N \to S \times N \) that moves firms over productivity and employment states, and there is an associated function \( q : (S \times N) \times (S \times N) \to [0, 1] \) that gives the probability of going from state \((s, n^-)\) to state \((s^+, n)\).

Figures 1 and 2 illustrate the employment policies. They have productivity \( s \) on the \( x \)-axis, past employment \( n^- \) on the \( y \)-axis, and optimal employment \( n(s, n^-, w, g) \) on the \( z \)-axis. It is

---

\(^7\)Although both \( Q \) and \( q \) also depend on \( w \) and \( g \), these arguments are omitted here and in the following.
clear that employment of continuing firms increases in \( s \) given \( n^- \). Moreover, for each \( n^- \), there is a neighborhood of \( n^- \) where employment is not changed, and is different from the firing-cost-free economy. Finally, when exiting firms are exempt from firing costs, they continue to employ \( n^- \) when exiting at the end of the period. As \( s \) passes the exit threshold \( s_x \), firms expect to continue, and do lay off workers.

Finally, it is clear that the value of incumbent firms is decreasing in the wage \( w \) and in the aggregate growth rate \( g \). The same holds for the employment policy, except that it is independent of \( g \) in the frictionless case.

**Exit:** Firms will exit if the expected value of continuing conditional on current states is less than that of exiting. Define the latter as

\[
V^x = -\mathbb{I}_x c^n w n = \begin{cases} 
0 & \text{if } c^n = 0 \lor \mathbb{I}_x = 0, \\
-c^n w n & \text{if } \mathbb{I}_x = 1.
\end{cases}
\]

(11)

This is constant in \( s \). Since the value function is strictly increasing in \( s \) for any \( n^- \), there is a unique threshold \( s_x \) where the expected value of continuing equals the value of exit. Firms will exit when they draw an \( s \) below this. Just as \( V \), \( s_x \) is a function of \( n^- \), \( w \), and \( g \) defined by

\[
s_x(n^-, w, g) = \{s | E[V(s^+, n, w, g)] = V^x \}.
\]

(12)

Taking into account the exit decision leads to a modification of the transition function to become

\[ Q_x : \bar{S} \times N \to (\bar{S} \cap \mathcal{S}) \times N, \]

where now the support of the productivity state \( s \) is partitioned into \( \bar{S} = \{s | s \geq s_x(n^-, w, g)\} \) (continue) and \( \mathcal{S} = \{s | s < s_x(n^-, w, g)\} \) (exit). The latter is an absorbing state. Note that the partition may differ across different elements of \( N \). The probability of going from \( (s, n^-) \in \bar{S} \times N \) to \( (s^+, n) \in (\bar{S}(N) \cap \mathcal{S}(N)) \times N \) then is given by a function \( q_x(\cdot) \).

The dependence of the exit threshold on the other variables is crucial for the selection effect. Clearly, \( s_x \) increases in \( w \) and \( g \). As the value function is weakly decreasing in \( n^- \), the exit threshold is weakly increasing in it. Finally, with no firing costs upon exit, the value of exit is higher, and the exit threshold will therefore also be higher. In this sense, exempting exiting firms from firing costs provides incentives to exit; and these will particularly affect low-productivity firms. By improving the distribution of surviving firms, this can boost growth, as shown below.

**Entry:** Potential entrants will enter if the expected net value of doing so is non-negative. So in equilibrium, the free entry condition

\[
E[V^e(s^0, w, g)] = c^e
\]

(13)
holds. Since the distribution of $s^0$ and $c^e$ are exogenous features of technology, this equation will pin down the wage, given a growth rate. If the wage was below (above) its equilibrium value, there would be additional (less) entry, driving up (down) the wage.

All firms’ decisions combined and the process for idiosyncratic shocks yield the law of motion for the firm productivity distribution $\tilde{\mu}(\cdot)$

$$\tilde{\mu}(s^+, n) = \int_N \int_{\bar{S}(N)} (1 - \delta) \tilde{\mu}(s, n^-) q_x(s^+, n|s, n^-) \, ds \, dn^- + \eta(s^+) \mathbb{1}\{n = n(s^+, 0, w, g)\}. \quad (14)$$

The integral describes the motion of incumbents. Exit is captured by the transition function $q_x(\cdot)$ and by the restriction of the domain of the integral to $N \times \bar{S}(N)$. Entry is given by $\eta(\cdot)$. It only contributes to $\tilde{\mu}(s^+, n)$ for $n$ that are optimal for entrants, given $s^+$, $w$ and $g$, as indicated by the indicator function. For later use, also denote the integral by $\tilde{\mu}'$. Discretizing the state space and denoting the identity matrix by $I$, the ergodic firm distribution can easily be obtained from the law of motion as $\tilde{\mu} = (I - Q_x')^{-1} \eta$ in the case without firing costs, and by iteration on the law of motion in the case with firing costs.

### 3.2 Equilibrium Definition

Define a stationary competitive equilibrium of the stationarized economy as real numbers $w$, $g$ and $M$, functions $n(s, n^-, w, g)$, $V(s, n^-, w, g)$, and $s_x(n^-, w, g)$, and a probability distribution $\mu(s, n^-)$ such that:

1. Consumers choose consumption, asset holdings, and labor supply optimally, so the interest rate is given by equation (9);
2. all active firms choose employment optimally according to the employment policy $n(\cdot)$, yielding value $V(\cdot)$ as described by equation (10) for all $(s, n^-, w, g)$;
3. exit is optimal: $s_x(\cdot)$ is given by equation (12) and firms exit if they draw an $s < s_x(\cdot)$, given $n^-$, $w$ and $g$;
4. entry is optimal and free: given a distribution $f_{s^0}$ over entrants’ productivities $s^0$, an entry cost $c^e$, $w$ and $g$, firms enter until the net value of entry equals its cost (equation (13));
5. the labor market clears: given $w$ and $g$, aggregate labor demand $M \int_N \int_{\bar{S}} \mu(s, n^-) \, n(s, n^-, w, g) \, ds \, dn^-$ equals supply as chosen by households;
6. given $g$, the firm distribution evolves according to the law of motion given by equation (14); and
The firm distribution is stationary: $\mu(s, n^-) = \mu^+(s, n^-)$ for all $(s, n^-)$.

The last condition also implies that the law of motion of the firm distribution yields the same growth rate as the one that agents take as given in the decisions that generate this law of motion.

Existence of equilibrium for similar economies is proven e.g. by Hopenhayn (1992); the proof here would proceed along very similar lines.

3.3 Algorithm for finding equilibrium

Equilibrium values depend on the parameter $\kappa$ that determines the relationship between entrants’ and incumbents’ productivity distributions. Its value for the benchmark economy will be imputed in the calibration in Section 4 below. Given a $\kappa$, equilibrium can then be calculated for any changes to that economy, such as the introduction of firing costs.

The numerical implementation is as follows. The state space $S \times N$ is discretized into a grid of $100 \times 800$ points. Using more points does not significantly affect results. The $N$ grid is chosen such that it contains the optimal employment quantities chosen by a firm in the frictionless economy for the points in $S$. Firm value can be obtained by value function iteration for each $(s, n^-)$ pair given $g$ and $w$. This also yields the exit thresholds $s_x(n^-, w, g)$ as defined in equation (12), and the transition function $Q_x$, given $g$ and $w$. For any fixed $g$, equation (13) determines the equilibrium wage $w$, and thereby the exit threshold and transition function for that $g$. Using these, the ergodic firm productivity distribution can be obtained; in the frictionless case directly as $\tilde{\mu} = (I - Q'_x)^{-1} \eta$ and in the case with firing cost by iteration on the law of motion for $\mu$ (equation (14)). The correct $g$ then is the one consistent with Assumption 2. Since $\hat{\kappa} = s_{t}^{\text{max}} - \hat{s}_{it}$ declines monotonically in $g$, this can be achieved rather quickly.

3.4 The growth rate

The growth rate $g$ is driven by the selection process and by the distance $\kappa$ between entrants’ and incumbents’ mean productivity. Intuitively, what happens is the following. In the growing economy, the productivity of incumbents follows a random walk. This implies that for a given set of firms, each firm’s productivity is constant in expectation, but the variance of those firms’ productivity distribution grows over time. However, with exit, the exit threshold truncates the firms’ productivity distribution from below. As a result, the distribution can only expand upwards, and average productivity of this set of firms grows. Hence, selection drives growth. However, as time goes by, firms keep on exiting, and the distribution thins out. (In a way, this process is similar to the one in Jovanovic (1982).) This is why entry is needed to sustain growth:
In a stationary equilibrium (of the stationarized economy), the measure of firms is constant, and exiting firms are replaced by entering ones. Yet while exiting firms are at the bottom of the distribution, entering firms are more productive – otherwise they would not enter. This results from comparing the exit condition \( E_{12} \) and the entry condition \( E_{13} \). Moreover, they enter at a constant distance from the productivity frontier by Assumption \( E_2 \). As a result, the productivity distribution shifts to the right: the bottom firms are replaced by more productive entrants, while some firms in the upper part of the distribution are lucky, receive positive shocks, and more that part of the distribution to the right.

For a more formal analysis, define the growth rate \( g \) as the difference between average (log) productivity in a period and the subsequent period:

\[
g \equiv E\hat{\mu}^+ - E\hat{\mu}.
\]  

(15)

Now and in the following, in a slight abuse of notation, the expectation of a random variable is denoted by the expectation of its distribution, i.e. writing \( E\mu \) instead of \( E(s), s \in \mu \). Consider the growing economy on its balanced growth path. Using the law of motion of \( \mu \), the growth rate can then be written as

\[
g = (1 - e) [E(Q_x \mu) - E\mu] + e (s^\text{max} - \kappa - E\mu),
\]  

(16)

where \( e \) is the entry (= exit) rate. (This is for the frictionless case, for the sake of simplicity. The reasoning carries over to the case with firing costs, at the cost of significantly more complicated notation.) The first term in the sum gives the effect of selection, i.e. the difference between the mean productivity of surviving firms and that of all firms present in the preceding period. The tougher market selection is, and the more firms at the low end of the productivity distribution exit, the larger this term. So it increases in \( s_x \). This also makes it clear that the growth rate and welfare do not have to behave in the same way; in the extreme, eliminating all but the most efficient firms would imply a high \( g \), but harm welfare due to decreasing returns to scale at the firm level and the direct cost of turnover (financing a lot of entry every period). The second term in (16) is the effect of entry, or imitation. It is clear, as suggested above, that it decreases in \( \kappa \). Entry affects the selection process in the sense that a more dispersed distribution of entrants in the previous period (higher \( \sigma_x^2 \)) will increase \( E(Q_x \mu) \), while in the entry period itself, entry has a negative effect if \( \kappa \) is such that \( s_t^\text{max} - \kappa < E\mu \). So the full positive effect of entry is not realized immediately, but only in combination with market selection in subsequent periods. For related reasons, the entry rate enters in two ways. By increasing current entry, it lowers the
growth rate. But on the balanced growth path considered here, more entry also implies more exit, driving up the exit threshold and thereby average productivity of surviving firms, so its net effect is ambiguous. Its contribution is more positive the smaller $\kappa$ in absolute value.

The growth rate and turnover: From the above, intuitive discussion, it is not obvious if the effect of firm turnover on the growth rate is always positive. To derive this, first express the growth rate as

$$g = (1 - e)(EQ\mu_{cont} - E\mu_{cont}) + e(\mu^+ - E\mu_{exit})$$

$$= e(\mu^+ - E\mu_{exit}),$$

(17)

where the firm distributions of the growing economy are used, and $\mu_{cont}$ and $\mu_{exit}$ refer to the distributions of continuing and exiting firms, respectively. The second equality holds because the expectation of the untruncated log productivity process is constant. The expression shows very neatly that the growth rate is positive if on average, entrants are more productive than exiting firms. Assume now that this is the case.

The derivative with respect to the exit threshold is

$$\frac{\partial g}{\partial s_x} = e\left(\frac{\partial E\eta^+}{\partial s_x} - \frac{\partial E\mu_{exit}}{\partial s_x} + \frac{\partial e}{\partial s_x}(\mu^+ - E\mu_{exit})\right).$$

(18)

The second term is positive since clearly, the entry and exit rate increases in the exit threshold. The first term is also positive since changing the exit threshold shifts the probability mass of the firm distribution to the right. Hence, the part that entrants target (a subset of this distribution bounded below by some value of productivity $s_x$) gains mass compared to the part where exiting firms are located.

3.5 Optimality

Growth in this economy is driven by selection among surviving firms and a spillover to entering firms. In the competitive equilibrium, firms do not take this into account. In particular, in their exit decision, firms do not take into account how their decision to exit or to remain active influences the productivity of entering firms. Therefore, a social planner could improve upon the competitive equilibrium by taking this into account. The following paragraphs provide a brief discussion about where the competitive equilibrium deviates from the optimal outcome,

---

8Note that equation (16) cannot be used directly for calculating the equilibrium since $Q_x$, and thereby $\mu'$, depends on $g$. This is why finding $g$ is a fixed-point problem.
and with which instruments a planner could implement the optimal outcome as a competitive equilibrium.

Suppose that there is a benevolent social planner that maximizes the representative agent’s utility. Further suppose that the planner faces the same technological constraints as firms in the competitive equilibrium, but can directly impose an exit threshold $s_x$ for all firms in the economy. This then determines the firm distribution and the growth rate. The planner also faces the decision of how much output to allocate to consumption, and how much to the construction and operation of firms.\footnote{Note that this tradeoff is reflected in the valuation of firms: Their social value is in terms of the contribution of their output to the representative agent’s utility, i.e. in terms of marginal-utility weighted output, not profits. However, the marginal utility terms drop out in all equations, so firm value is again in terms of output below. This also makes the comparison with competitive equilibrium conditions easier.}

The choice of an exit threshold aims at obtaining the best firm distribution and growth rate, given the associated cost of firm turnover. (More exit implies more selection and faster growth as discussed above, but also higher costs of financing entry investment.) This implies maximizing the value of a portfolio of firms, minus the social cost of turnover, i.e.

$$
\max_{s_x} \int \mu(s)V(s)\,ds - c^e = \max_{s_x} \int S (Q_x\mu)(s)V(s)\,ds + e \int \eta(s)V(s)\,ds - ec^e \tag{19}
$$

by choosing $s_x$. (Remember that $\bar{S}$ denotes the set of continuing firms, i.e. $\bar{S} = \{s| s \geq s_x(n^-, w, g)\}$.) In this objective, $\bar{S}, Q_x\mu, e,$ and firm value $V$ all depend on $s_x$. In the stationarized formulation of the model, this yields the first order condition

$$
-(Q_x\mu)(s_x)V(s_x) + \int_{\bar{S}} \frac{\partial Q_x\mu}{\partial s_x} V(s)\,ds + \int_{\bar{S}} \mu \frac{\partial E[V]}{\partial s_x}\,ds + \int_{\bar{S}} \frac{\partial e}{\partial s_x} (EV^e - c^e) = 0. \tag{20}
$$

The planner’s investment vs consumption decision yields a condition that is analogous to the free entry condition in the competitive equilibrium:

$$
c^e = EV^e. \tag{21}
$$

Inserting (21) into (20) and using $Q_x V(s, \cdot) = E[V(s^+, \cdot)|s]$ yields the simplified condition

$$
\mu(s_x)E[V(\cdot)|(s_x)] = \int_{\bar{S}} \frac{\partial Q_x\mu}{\partial s_x} V(s)\,ds + \int_{\bar{S}} \mu \frac{\partial E[V]}{\partial s_x}\,ds. \tag{22}
$$

Compare this to the competitive equilibrium exit condition (12). There, firms exit if their expected value is negative (or smaller than the value of exit, $V^e$). The planner, in contrast, takes into account the impact of the exit decision on the remaining distribution (first term on the right-hand side (RHS)) and on the value of other firms (second term). As seen above, the
former is positive. The latter is positive for low $s_x$ and negative for large ones, for the same
reason as there is a unique $s_x$ in the competitive equilibrium: Firm value is negative for low $s$
due to the fixed cost; and the option to exit is valuable in that situation. Hence, there is a range
of $s_x$ for which the RHS is positive, and it may become negative for high $s_x$.

To evaluate if the competitive equilibrium is optimal, insert two equilibrium conditions from
there into equation (22). First, the first order condition for the choice of $s_x$ (not given above)
implies that $\partial E[V]/\partial s_x = 0$, hence the second term on the RHS of (22) is zero.\footnote{This can be seen from rewriting the firm’s exit problem. Above, we wrote it as $\max_{s_x} (E[V|s_x], V^x)$. Now let $V^*(s, s_x)$ be the firm value if the exit rule is given by some fixed $s_x$. Then the problem becomes $\max_{s_x} E[V^*]$, with the attached FOC $\partial E[V^*]/\partial s_x = 0$. (The objective is concave, so the FOC is also sufficient.) Since $V^*(s, s_x)$ for $s_x$ that solves the FOC is the same as the firm value function given in (10), this condition is exactly the same as the one used in the text.} Second, the
left-hand side (LHS) of equation (22) equals $\mu(s_x)V^x$ by equation (12). The fact that the first
term on the RHS of (22) is positive, i.e. the effect of selection, now implies that the LHS should
also be positive to achieve a social optimum, i.e. $V^x > 0$. Since $V^x = 0$ in the competitive equilibrium without firing cost or with $I_x = 0$, and $V^x < 0$ when $I_x = 1$, the competitive equilibrium is not optimal. Charging firing costs to exiting firms – an exit tax – is even worse, as detailed in Section 5.

This reasoning also shows how the optimal allocation can be implemented as a competitive equilibrium: by an exit subsidy (so $V^x > 0$) that makes (22) hold with equality. Since $V$ is
continuous and monotonic in $s$, every $s_x$ can be achieved in (eq: sx) by setting the right $V^x$.
In the benchmark calibration, the subsidy needed equals 14% of the entry investment $c^e$, and
the resulting allocation yields 2.6% higher welfare (neglecting the issue of how to finance the subsidy).

Finally, an exit subsidy is not the only instrument that could be used to affect the exit
threshold. Anything that reduces firm value (such as a lump-sum tax, or EPL) or otherwise
affects firm turnover (such as an entry subsidy) could be used for that aim. The advantage
of the exit subsidy is that it affects only the exit threshold and hence constitutes the cleanest instrument.

4 Benchmark Economy

To derive quantitative conclusions, the model has to be calibrated. More fundamentally, as seen
in Section 3.4, the equilibrium growth rate $g$ depends on the unknown technology parameter
$k$. That parameter cannot be inferred directly from evidence on the relative productivity of
entrants since in empirical work, relative productivity of entrants usually is measured several years after entry and conditional on survival. \( \kappa \), on the other hand, represents the unconditional relative productivity of potential but unrealized projects. A way around this is to find a \( \kappa \) that is consistent with findings on the relative productivity of entrants after selection through the entry process. Hence I proceed as follows: calibrate the economy such that the relative productivity of entrants exactly matches the data moment, and then infer \( \kappa \) from its definition in equation (5).

Data moments used in calibration refer to the US non-farm business sector. According to the World Bank’s Doing Business database, firing costs are zero in the US, and the ”Difficulty of Firing” index and other measures of employment protection are among the lowest worldwide. This suggests to use it as the no-firing cost benchmark.

As usual, I calibrate the model by using commonly used values from the literature for some baseline parameters, and choosing the remaining ones such that the distance between a set of model moments and corresponding data moments is minimized, where distance is the mean squared relative deviation. The fact of dealing with distributions leads to some practical difficulties. First, to obtain model moments, the whole model has to be solved for each parameter combination under consideration. Second, the distance between model and data moments is a highly nonlinear function of the parameters with many local minima. To find the global minimum, a genetic algorithm as laid out in Dorsey and Mayer (1995) is used.

The parameter values adopted from the literature are 0.64 for the labor share \( \alpha \) and 0.95 for the discount factor \( \beta \). The disutility of labor \( \theta \) is set such that labor force participation fits the value of 66\% reported by the BLS and the ILO. The five parameters that remain to be assigned are the variance of the log productivity distribution of entrants \( \sigma^2_e \), the variance of the the idiosyncratic productivity shock hitting incumbents \( \sigma^2_r \); the fixed operating cost \( c^f \), the entry cost \( c^e \); and the breakdown probability \( \delta \). In addition, the value of \( \kappa \) has to be inferred as described above. The data moment used for that is the relative productivity of entrants. Foster et al. (2001) report this to be 99\% of that of all active firms, counting as entrants firms that entered within the last ten years and are still active.

The remaining parameters are chosen to match three moments referring to the entire economy, and two moments related to entry and post-entry behavior: the job turnover rate, average plant size, dispersion of the productivity distribution, the four-year survival rate of entrants, and the share of aggregate productivity growth due to entry and exit. These moments are chosen because each captures a different aspect of the firm distribution and its dynamics and therefore
allow a relatively full description. Average plant size and the dispersion of the productivity distribution are closely related to the mean and variance of the firm productivity distribution. The job turnover rate describes its dynamic behavior. The survival rate of entrants indicates the severity of the selection process, and the last moment fixed the importance of the entry and selection process at a realistic value.

The job turnover rate is the sum of job creation and job destruction at continuing, entering and exiting plants in a year, divided by total employment in that year; it is a crucial dynamic feature of the plant distribution. According to the BLS, it is 28% yearly in the US. Cross-country differences in this variable are significant, as documented by Davis, Haltiwanger and Schuh (1996). The US value is on the high side among developed economies.

Average plant size (employment) provides a measure of the mean of the firm distribution. Bartelsman et al. (2004) (BHS) report it to be 26.4 for the whole U.S. economy. The dispersion of the productivity distribution helps pin down the variance of the incumbents’ productivity shock. For lack of better data, I use the measure from Dwyer (1998) who finds that for the U.S. textile industries, the average ratio between the 85th and the 15th percentile of the plant productivity distribution is 3. Other studies report results in the same ballpark for other countries and industries (see e.g. Roberts and Tybout 1996).

Next, a crucial statistics describing the post-entry process is the survival rate of entrants. Matching it well is important for obtaining a good estimate of κ since the latter is calculated using the relative productivity of surviving entrants. The four-year survival rate, i.e. the proportion of entrants of a given year still active four years later, is 63% in the U.S. (BHS). This is lower than in most other industrialized countries, but higher than in many Latin American ones, though quantitatively, cross-country differences are not very large.

Finally, I aim to match the contribution of entry and exit to aggregate productivity. This is important for giving the right importance to the process of entry and selection relative to within-firm productivity growth. FHK find its value to be 26% for the U.S. manufacturing sector, and higher in retailing. Other studies find similar estimates, BHS give an overview. Calibration targets, their values for Germany (where available), and model values are given in Table 1. Adopted parameter values are given in Table 2. Model statistics fit all targets closely.

The calibration fits rather well even in dimensions that were not targeted. The firm turnover rate is very low, but its employment-weighted counterpart fits almost exactly. The discrepancy might arise if the model with its rather high entry cost does not capture a fringe of very small, short-lived firms. BHS show that these are numerous. This does not seem to matter too much
Table 1: Calibration: Model statistics, Targets (U.S.), values for Germany, all data for 1990s

<table>
<thead>
<tr>
<th>Statistic</th>
<th>model</th>
<th>U.S.</th>
<th>GER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average firm employment</td>
<td>26.4</td>
<td>26.4</td>
<td>17</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>66.4%</td>
<td>66%</td>
<td>56.7%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>99%</td>
<td>99%</td>
<td></td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>28.0%</td>
<td>28%</td>
<td>16.5%</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>2.9</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>61.9%</td>
<td>63%</td>
<td>68%</td>
</tr>
<tr>
<td>Share of aggregate productivity growth due to entry and exit</td>
<td>27.2%</td>
<td>26%</td>
<td></td>
</tr>
<tr>
<td>not used in calibration:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>1.85%</td>
<td>1.80%</td>
<td>1.65%</td>
</tr>
<tr>
<td>Employment-weighted firm turnover</td>
<td>7.2%</td>
<td>7.0%</td>
<td>3.9%</td>
</tr>
<tr>
<td>Firm turnover rate</td>
<td>13.1%</td>
<td>22%</td>
<td>13%</td>
</tr>
<tr>
<td>Seven-year growth rate of entrants</td>
<td>39%</td>
<td>40%</td>
<td>25.4%</td>
</tr>
</tbody>
</table>


for firm growth and aggregate growth, though, since the calibrated model matches the seven-year growth rate of surviving entrants well. Most remarkably, the aggregate growth rate is very close to that found in the data – a clear indication that the growth mechanism proposed here and its calibration are plausible.

5 Firing costs and productivity growth

The focus of the paper is the analysis of the impact of firing costs on aggregate productivity growth. Since growth is endogenous in the model developed above, frictions can affect not only the level (as in previous literature), but also the growth rate of output and productivity. This section will explore their effect first theoretically, then empirically.

5.1 Theoretical discussion

It is crucial to note that firing costs affect firms in two ways: they constitute a friction to the adjustment of labor, and they are a tax on exit, if charged to exiting firms. Their effects can most easily be seen in the light of the discussion of optimality in Section 3.5. Firstly, as an adjustment friction, firing costs cause firms’ employment to deviate from optimal employment in the frictionless economy. This lowers firm value and the incentive to enter or to continue in
Table 2: Calibration: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.64</td>
<td>Labor share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.95</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.15</td>
<td>Disutility of working</td>
</tr>
<tr>
<td>$\sigma_c^2$</td>
<td>0.60</td>
<td>Variance of log productivity distribution of entrants</td>
</tr>
<tr>
<td>$\sigma_e^2$</td>
<td>0.1</td>
<td>Variance of idiosyncratic productivity shock</td>
</tr>
<tr>
<td>$c_f$</td>
<td>3.3%</td>
<td>Fixed operating cost, % of avg firm output</td>
</tr>
<tr>
<td>$c_e$</td>
<td>198%</td>
<td>Cost of entry, % of avg firm output</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0275</td>
<td>Probability of exogenous exit</td>
</tr>
<tr>
<td>$\exp(\kappa)$</td>
<td>6.04</td>
<td>Ratio prodty best incumbents/avg entrant</td>
</tr>
</tbody>
</table>

operation. As a consequence, the exit threshold increases and comes closer to the socially optimal one that solves equation \([22]\), but at the cost of distorting firms’ labor demand policy. Hence, firing costs are a very “dirty” instrument for achieving stricter selection. Secondly, if charged upon exit, firing costs are a tax on exit. They then provide an incentive towards continuing, and remove the exit threshold further from its optimal value. The next paragraphs provide a more detailed discussion of the effect of firing costs.

Consider first the case where firing costs are only charged to continuing firms ($1_x = 0$). Then, they do not affect the value of exit $V^x$ compared to the benchmark economy. They do reduce the value of continuing, though. For given $g$, equation \([13]\) then implies a lower equilibrium wage $w$. This keeps the expected value of entry equal to $c^e$, and the exit threshold for entrants constant. Entrants are also the only firms with $n^- = 0$. This is because it is never optimal for previously active firms to choose an employment level of zero (not even in the frictionless case). However, firm value falls in $n^-$, and the exit threshold rises in it. Hence, the average exit threshold in the economy with firing cost will be at least as high as in the frictionless economy. Given $g$, higher $s_x$ implies a higher exit rate and more severe selection. If now the selection effect dominates the entry effect in equation \([16]\), the growth rate will rise, as was the case for all parameterizations explored; otherwise it will fall. For all equilibrium conditions to hold, $w$ has to fall more with the higher growth rate than if $g$ stayed constant. The lower wage and costless firing upon exit imply that average employment rises, and the number of firms falls. Just as the wage, stationarized consumption declines. The size of this fall relative to the higher growth rate determines the sign of the reaction of welfare.

Charging the firing cost also to exiting firms complicates the picture. First of all, in this
case, too, the stationarized equilibrium wage has to be lower than in the benchmark for given $g$.
The effect on exit is more complicated because now both the value of continuing and the value of exit are lower in equation (12), and both fall in $n^-$.
However, since exiting implies bearing firing costs immediately, $V^x$ drops by more than the value of continuing, and $s_x$ tends to fall, implying less selection and lower growth.
Because of the fall in the growth rate, the wage need not fall as much as for constant $g$.
Average firm size in this case drops despite the fall in wages because firing costs discourage hiring.
As a result, the production structure is less efficient than in the benchmark economy since firms have to cover the fixed cost anyway, and stationarized output declines.
Hence, welfare is unambiguously lower in this case.

5.2 Quantitative evaluation

This section reports quantitative results on the effect of altering the benchmark economy by introducing firing costs of $c^n$ times the equilibrium wage for each worker fired. $c^n$ is set to one, i.e. a year’s wages.
This is close to the average over continental European countries according to the World Bank’s Doing Business Database.

Results are reported in Table 3 and fit the qualitative patterns described above.
Note that consumption is a stationarized value that cannot be compared directly across columns.
To properly evaluate welfare, the welfare loss using the equivalent variation is given.
The number indicates what percentage of consumption would need to be taken away from consumers in the no-firing-cost case to make their welfare equivalent to that of consumers in each of the other two economies.

The most salient result are the changes in growth rates.
Introducing firing costs decreases the growth rate by around 1 tenth of a percentage point when firing costs are always charged.
When exiting firms are exempt, the growth rate rises marginally.
For the rest, results fit the qualitative results outlined above.
Welfare clearly drops with firing costs.
For $\Pi_x = 0$, the distortion in the allocation of labor outweighs the higher growth rate, implying a welfare loss of 3.8%.
For $\Pi_x = 1$, the growth rate actually drops, so the welfare loss is even larger at 5.4%.

It is well-known that the variance of idiosyncratic shocks is larger in the service sector.
For instance, the coefficient of variation of firm size is up to three times as high as in manufacturing sectors (BHS), job turnover is higher (see e.g. Davis, Faberman and Haltiwanger 2006), and firm turnover is higher (BSS).
In such a setting, employment protection legislation has a more constraining effect on firms.
And indeed, recent growth differences between Europe and the US were largest in a large subsector of the service sector, namely in ICT-using services (van Ark
Table 3: Results: Introducing firing costs (always: \( \mathbb{1}_x = 1 \), exit exemption: \( \mathbb{1}_x = 0 \), benchmark economy = 100)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Benchmark</th>
<th>( \mathbb{1}_x = 1 )</th>
<th>( \mathbb{1}_x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm turnover rate</td>
<td>13.1%</td>
<td>13.3%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>28.0%</td>
<td>12.1%</td>
<td>12.3%</td>
</tr>
<tr>
<td>Average firm size</td>
<td>26.4</td>
<td>29.4</td>
<td>30.7</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>61.9%</td>
<td>58.7%</td>
<td>57.9%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>99%</td>
<td>98.7%</td>
<td>95.9%</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>66.4%</td>
<td>65.3%</td>
<td>66.5%</td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>1.848%</td>
<td>1.763%</td>
<td>1.851%</td>
</tr>
<tr>
<td>Consumption (stationarized)</td>
<td>100</td>
<td>94.8</td>
<td>96.2</td>
</tr>
<tr>
<td>Welfare change</td>
<td>-5.4%</td>
<td>-3.8%</td>
<td></td>
</tr>
</tbody>
</table>

(Equivalent variation, % of \( c \))

et al. 2003, Blanchard 2004). Table 4 shows the effect of firing costs of a year’s wages in an economy where \( \sigma^2 \), the variance of the idiosyncratic shock, is raised from 0.15 to 0.1 to mimic the service sector. First note that this sector has a higher growth rate than the benchmark economy – this is the positive effect of \( \sigma^2 \) on the growth rate alluded to before. Turnover rates and productivity dispersion are higher, too. Now firing costs have a stronger effect on the growth rate and on welfare for both settings of \( \mathbb{1}_x \). If firing costs are only charged to continuing firms (\( \mathbb{1}_x = 0 \)), there is stricter selection, and the growth rate rises by 0.2%. If they are also charged to exiting firms (\( \mathbb{1}_x = 1 \)), the growth rate drops by 0.3%. Welfare falls in both cases.

Hence, firing costs can have potentially large growth rate effects, particularly in volatile sectors such as services. This fits very well with the pattern of recent growth rate differences between the US and Continental Europe. It also results that details of EPL regimes matter. and that dealing with exit efficiently is an important policy concern in its own right. In fact, charging firing costs to exiting firms does not reduce job turnover by much, but has potentially large additional welfare costs compared to charging them only to continuing firms.

One additional, but nontrivial, step in the analysis would be desirable: the model developed here completely abstracts from benefits of firing costs. Alvarez and Veracierto (2001) find that severance payments, which affect firms in a way similar to firing costs, are welfare-improving in a world with search frictions because agents become unemployed less often. This more than

\[ \text{This parametrization is preliminary.} \]
Table 4: Results: Service sector (always: $I_x = 1$, exit exemption: $I_x = 0$, benchmark economy = 100)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Benchmark</th>
<th>$I_x = 1$</th>
<th>$I_x = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm turnover rate</td>
<td>18.9%</td>
<td>16.2%</td>
<td>22.2%</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>39.6%</td>
<td>16.6%</td>
<td>17.8%</td>
</tr>
<tr>
<td>Average firm size</td>
<td>21.4</td>
<td>21.3</td>
<td>25.4</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>3.3</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>57.2%</td>
<td>56.9%</td>
<td>54.1%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>99%</td>
<td>101%</td>
<td>95.4%</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>66.4%</td>
<td>64.2%</td>
<td>67.0%</td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>3.9%</td>
<td>3.6%</td>
<td>4.1%</td>
</tr>
<tr>
<td>Consumption (stationarized)</td>
<td>100</td>
<td>95.1</td>
<td>91.4</td>
</tr>
<tr>
<td>Welfare change</td>
<td>-8.1%</td>
<td>-5.4%</td>
<td></td>
</tr>
</tbody>
</table>

(=equivalent variation, % of $c$)

compensates the static distortions. However, this result relies on an analysis where firing costs do not affect growth. Combining results obtained here with those by Alvarez and Veracierto allows the conjecture that in a model with growth and search frictions, firing costs would very probably be harmful when always charged, since they would decrease growth on top of the static distortions, but might still be welfare-improving if not charged upon exit.

6 Conclusion and directions for further research

This paper has analyzed the effect of firing costs on productivity growth, a topic that is currently receiving much attention in policy circles, notably in Europe, but has not been subject of much study in the theoretical literature. To perform the analysis, a model of growth through selection and experimentation has been developed, taking into account recent evidence on firm dynamics, particularly on the importance of job turnover, firm heterogeneity, and the contribution of entry and exit to aggregate productivity growth. In the model, firms receive idiosyncratic productivity shocks and therefore differ in productivity and employment. Growth occurs endogenously due to selection among incumbents, and due to imitation by entrants. In a nutshell, selection eliminates the worst active firms, so that entrants direct their imitation efforts towards the remaining, better ones. Modeling mean productivity of entrants as a constant fraction of the productivity frontier, the model economy grows through rightward shifts of the firm productivity distribution.

This environment of rich firm dynamics is well-suited to the analysis of firing costs. Here lies
the contribution of the paper. Besides inducing a misallocation of labor, reducing firm value, and discouraging entry, as in previous models, they will also discourage exit of low-productivity firms in this setup. This congests the selection process and slows down growth – the first main result of the paper. Exempting exiting firms from firing cost, however, speeds up the exit of inefficient firms and thereby growth – the second main result.

The model is calibrated to the US economy for a quantitative evaluation. The US is generally thought to feature low firing costs, so it can serve as a benchmark. Then the effects of introducing firing costs of one year’s wages are explored. Results show that firing costs always reduce welfare. Although they promote selection and growth when only charged to continuing firms, the misallocation of labor they induce outweighs this. When they are also charged to exiting firms, growth slows, and the welfare loss is even larger. In highly volatile environments such as the service sector, firing costs have an even stronger effect on growth, fitting evidence on growth differences between Europe and the U.S. in the last decade.

These results imply that EPL can matter for productivity growth. Moreover, it is crucial how labor market policies affect efficient firm exit. Charging firing costs to exiting firms implies small reductions in job turnover, but large costs in terms of lower growth.
References


Figure 1: The employment policy function when firing costs are always charged ($\Pi_x = 1$)

Figure 2: The employment policy function with firing cost exemption upon exit ($\Pi_x = 0$)
Figure 3: Growth through right-shifts of the firm productivity distribution

Figure 4: Productivity distribution of entrants and incumbents, difference: $\kappa$